MINERAL SOLUTIONS

CHAPTER I<br>Part 1<br>Level: $1^{\text {st }}$ year LMD

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## 1. Chemical equilibrium

### 1.1. Acid-base equilibrium

We find three definitions: Arrhenius (1887) and Brønsted-Lowry (1923), as well as the model of Lewis (1923). In solution chemistry, the most suitable theory is that of Brønsted-Lowry.

## a. Definition of Arrhenius (1887):

An acid: is a chemical substance capable of releasing $\mathrm{H}^{+}$ions (protons) in aqueous solution ( $\mathrm{H}_{3} \mathrm{O}^{+}$: oxonium or hydronium ion).

Example: $\mathrm{HA}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{A}^{-}+\mathrm{H}_{3} \mathrm{O}^{+} \quad\left(\mathrm{HCl}, \mathrm{CH}_{3} \mathrm{COOH}, \ldots.\right)$
A base: is a chemical substance capable of releasing $\mathrm{OH}^{-}$(hydroxide) ions in aqueous solution Example: $\mathrm{BOH}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{B}^{+}+\mathrm{OH}^{-} \quad(\mathrm{NaOH}, \mathrm{KOH}, \ldots \ldots)$

## b. Definition from Bronsted and Lowry (1923):

An acid: is a chemical species capable of releasing one or more $\mathrm{H}+$ protons; it is a proton donor.

Example: $\mathrm{HA}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{A}++\mathrm{H}_{2} \mathrm{O}\left(\mathrm{HCl}, \mathrm{CH}_{3} \mathrm{COOH}, \ldots.\right)$
A base: is a chemical species capable of capturing one or more $\mathrm{H}^{+}$protons; it is a proton acceptor.

Example: $\mathrm{A}^{-}+\mathrm{H}+\rightleftharpoons \mathrm{HA}$

## c. Definition from Lewis (1923):

An acid: is a body that accepts a pair of electrons
Example: $\mathrm{H}_{2} \mathrm{O}+\mathrm{H}^{+} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}$(appearances of a dative bond).
A base: is a body donating a pair of electrons
Example: $\mathrm{NH}_{3}+\mathrm{H}^{+} \rightleftharpoons \mathrm{NH}_{4}{ }^{+}$(appearances of a dative bond).

### 2.2. Monoacid and mono-base:

According to Bronsted, an acid is a species capable of releasing an $\mathrm{H}^{+}$proton to give it to another species, and a base is a species capable of capturing a proton coming from another species.

$$
\mathrm{Acid} \rightleftharpoons \mathrm{Base}+\left(\mathrm{H}^{+}+\mathrm{H}_{2} \mathrm{O}\right)
$$

The acid and the base are said: "Conjugates); they form an acid-base couple noted: Acid/Base).

## Example:

$\mathrm{CH}_{3} \mathrm{COOH}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{CH}_{3} \mathrm{COO}^{-}+\mathrm{H}_{3} \mathrm{O}^{+} ; \quad\left(\mathrm{CH}_{3} \mathrm{COOH} / \mathrm{CH}_{3} \mathrm{COO}^{-}\right)$
The acetate ion $\mathrm{CH}_{3} \mathrm{COO}^{-}$is the conjugate base of acetic acid $\mathrm{CH}_{3} \mathrm{COOH}$.

### 2.3. Poly-acids and Poly-bases:

A polyacid is a species capable of releasing 2 or more protons $\left(\mathrm{H}^{+}\right)$to give them to another species, and a base is a species capable of fixing 2 or more protons $\left(\mathrm{H}^{+}\right)$from another species.

Example: $\mathrm{H}_{2} \mathrm{SO} 4: \quad \mathrm{H}_{2} \mathrm{SO}_{4}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{HSO}_{4}^{-}+\mathrm{H}_{3} \mathrm{O}^{+}$

$$
\mathrm{HSO}_{4}^{-}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{SO}_{4}^{-2}+\mathrm{H}_{3} \mathrm{O}^{+}
$$

Example: $\mathrm{S}^{-2}: \quad \mathrm{S}^{-2}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{HS}^{-}+\mathrm{OH}^{-}$

$$
\mathrm{HS}^{-}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{2} \mathrm{~S}+\mathrm{OH}^{-}
$$

### 2.4. An ampholytes solution:

An amphoteric (or ampholyte) is a chemical species that can behave either as an acid or as a base. The corresponding solutions are called "Amphoteric).

## Example 1: $\mathrm{H}_{2} \mathrm{O}$

Couple 1: $\quad\left(\mathrm{H}_{3} \mathrm{O}^{+} / \mathrm{H}_{2} \mathrm{O}\right): \mathrm{H}_{3} \mathrm{O}^{+} \rightleftharpoons \mathrm{H}_{2} \mathrm{O}+\mathrm{H}^{+}$
Couple 2: $\quad\left(\mathrm{H}_{2} \mathrm{O} / \mathrm{OH}^{-}\right): ~ \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}^{+}+\mathrm{OH}^{-}$
Total reaction: $2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}$(Auto-protolysis reaction)
Example 2: $\mathrm{H}_{2} \mathrm{~S}$
Couple 1: $\left(\mathrm{H}_{2} \mathrm{~S} / \mathrm{HS}^{-}\right): \mathrm{H}_{2} \mathrm{~S} \rightleftharpoons \mathrm{HS}^{-}+\mathrm{H}^{+}$
Couple 2: $\left(\mathrm{HS}^{-} / \mathrm{S}^{-2}\right): \quad \mathrm{HS}-\rightleftharpoons \mathrm{S}^{-2}+\mathrm{H}^{+}$
Total reaction: $2 \mathrm{HS}^{-} \rightleftharpoons \mathrm{H}_{2} \mathrm{~S}+\mathrm{S}^{-2}$

### 2.5. Acid-base reaction:

An acid-base reaction is a proton transfer reaction between 2 couples: Acid 1/Base 1 and Acid2/Base2

Acid $1 /$ Base 1 : Acid $1 \rightleftharpoons$ Base $1+\mathrm{H}_{3} \mathrm{O}^{+}$
Acid 2/Base 2: Base $2+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons$ Acid 2

Total reaction: Acid $1+$ Base $2 \rightleftharpoons$ Base $1+$ Acid 2

### 2.5.1. Equilibrium constant: water dissociation, acidity and basicity:

There are four equilibrium constants: $\mathrm{K}, \mathrm{Ke}, \mathrm{Ka}, \mathrm{Kb}$
a. Equilibrium constant of a reaction $(K)$ :

The equilibrium constant $K$ of any acid-base reaction is given by the law of mass action:

$$
\mathrm{a} A+\mathrm{bB} \rightleftharpoons \mathrm{c} \mathrm{C}+\mathrm{dD}
$$

$\mathbf{K}=\left([\mathrm{C}]^{\mathrm{c}} *[\mathrm{D}]^{\mathrm{d}}\right) /\left([\mathrm{A}]^{\mathrm{a}} *[\mathrm{~B}]^{\mathrm{b}}\right)$

Or [A], [B], [C] and [D]: are the concentrations of species A, B, C and D at equilibrium.

## b. Water dissociation constant (Ke):

The self-protolysis equilibrium of water: $\quad 2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}$

$$
\mathrm{K}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{H}_{2} \mathrm{O}\right]} \quad \mathrm{Ke}=\mathrm{K} \cdot\left[\mathrm{H}_{2} \mathrm{O}\right]=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot\left[\mathrm{OH}^{-}\right]
$$

Ke : ionic product of water: $\mathrm{Ke}=\mathrm{K} .\left[\mathrm{H}_{2} \mathrm{O}\right]=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]$
Calculate the concentration of water $\left[\mathrm{H}_{2} \mathrm{O}\right]$ :
At $\mathrm{t}=25 \mathrm{C}^{\circ}, \mathrm{Ke}=10^{-14} \Rightarrow \mathrm{pKe}=14$.
We have: $\mathrm{C}\left(\mathrm{H}_{2} \mathrm{O}\right)=\mathrm{n}\left(\mathrm{H}_{2} \mathrm{O}\right) / \mathrm{V}\left(\mathrm{H}_{2} \mathrm{O}\right)$, and $\rho\left(\mathrm{H}_{2} \mathrm{O}\right)=\mathrm{m}\left(\mathrm{H}_{2} \mathrm{O}\right) / \mathrm{V}\left(\mathrm{H}_{2} \mathrm{O}\right)=1 \mathrm{~g} / \mathrm{ml}\left(\right.$ or: $\left.1 \mathrm{~g} / \mathrm{cm}^{3}\right)$
We take: $\mathrm{m}\left(\mathrm{H}_{2} \mathrm{O}\right)=1 \mathrm{~g} \Rightarrow \mathrm{~V}\left(\mathrm{H}_{2} \mathrm{O}\right)=1 \mathrm{ml}$
$\left.\mathrm{n}\left(\mathrm{H}_{2} \mathrm{O}\right)=\mathrm{m}\left(\mathrm{H}_{2} \mathrm{O}\right) / \mathrm{M}\left(\mathrm{H}_{2} \mathrm{O}\right)=1 / 18=0.05555 \mathrm{~mol}\left(\mathrm{M}\left(\mathrm{H}_{2} \mathrm{O}\right)=2(1)+16=18 \mathrm{~g} / \mathrm{mol}\right)\right)$
$\mathrm{C}\left(\mathrm{H}_{2} \mathrm{O}\right)=0.0555555 / 1.10^{-3} \Rightarrow \mathrm{C}\left(\mathrm{H}_{2} \mathrm{O}\right)=55.55 \mathrm{~mol} / \mathrm{l}$.

## c. Acidity constant (Ka):

Weak acid: $\quad \mathrm{AH}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{A}^{-}+\mathrm{H}_{3} \mathrm{O}^{+} \quad$ (Couple: $\mathrm{HA} / \mathrm{A}^{-}$)
The equilibrium constant:

$$
\mathrm{K}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot\left[\mathrm{A}^{-}\right]}{\left[\mathrm{H}_{2} \mathrm{O}\right] \cdot[\mathrm{AH}]}
$$

The equilibrium constant of dissociation of the acid called "acidity constant Ka ":

$$
\mathrm{Ka}=\mathrm{K} \cdot\left[\mathrm{H}_{2} \mathrm{O}\right]=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot\left[\mathrm{A}^{-}\right]}{[\mathrm{AH}]}
$$

Knowing that: $\mathrm{pka}=-\log \mathrm{Ka}$

## Noticed:

Increasing acidity: $\mathrm{Ka} \Rightarrow \Rightarrow \mathrm{pKa} \searrow$
Increasing basicity: $\mathrm{Ka} \searrow \Rightarrow \mathrm{pKa} \triangle$

## d. Basicity constant (Kb):

Weak base: $\mathrm{B}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{BH}^{+}+\mathrm{OH}^{-}\left(\right.$Couple: $\left.\mathrm{BH}^{+} / \mathrm{B}\right)$
The equilibrium constant:

$$
\mathrm{K}=\frac{\left[\mathrm{BH}^{+}\right] \cdot\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{H}_{2} \mathrm{O}\right] \cdot[\mathrm{B}]}
$$

The equilibrium constant of acid dissociation called "acidity constant Kb ":

$$
\mathrm{Kb}=\mathrm{K} \cdot\left[\mathrm{H}_{2} \mathrm{O}\right]=\frac{\left[\mathrm{BH}^{+}\right] \cdot\left[\mathrm{OH}^{-}\right]}{[\mathrm{B}]}
$$

## Noticed:

The relationship between Ka and $\mathrm{Kb}: \quad \mathrm{Ke}=\mathrm{Ka} . \mathrm{Kb}$

### 2.5.2. $\mathbf{p H}$ (Hydrogen potential):

a. Definition of pH : the acidity of an aqueous solution depends on the concentration of $\mathrm{H}_{3} \mathrm{O}^{+}$ ions.

$$
\mathrm{pH}=-\log \mathrm{H}_{3} \mathrm{O}^{+} \quad\left(\mathrm{p}=-\log ; \quad \mathrm{H}=\mathrm{H}_{3} \mathrm{O}^{+}\right)
$$

## b. Relationship between $\mathrm{pH}, \mathrm{pKa}$ and concentrations:

Weak acid: $\operatorname{Acid}(\mathrm{AH})+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \operatorname{Base}\left(\mathrm{A}^{-}\right)+\mathrm{H}_{3} \mathrm{O}^{+}$(Couple: $\left.\mathrm{HA} / \mathrm{A}^{-}\right)$

$$
\mathrm{Ka}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot[\text { Base }]}{[\text { Acide }]}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot\left[\mathrm{A}^{-}\right]}{[\mathrm{AH}]}
$$

$\mathrm{Ka} \cdot[$ Acide $]=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \cdot[$ Base $] \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{Ka} \cdot[$ Acid $] /[$ Base $]$
$-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log (\mathrm{Ka} .[$ Acid $] /[$ Base $]) ; \quad\left(\mathrm{pH}=-\log \mathrm{H}_{3} \mathrm{O}^{+} ; \mathrm{pKa}=-\log \mathrm{Ka}\right)$.

$$
\mathrm{pH}=\mathrm{pka}+\log \frac{[\text { Base }]}{[\text { Acide }]} \quad \text { Relation d'Henderson }
$$

## The pH of the water:

The water auto-protolysis: $2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}$
In pure water the quantities of $\mathrm{H}_{3} \mathrm{O}^{+}$and $\mathrm{OH}^{-}$ions are equal $\Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{OH}^{-}\right]$
At: $\mathrm{t}=25 \mathrm{C}^{\circ} \mathrm{Ke}=10^{-14},\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]=\mathrm{Ke}=10^{-14} \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2}=\mathrm{Ke}=10^{-14}$
$\Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\sqrt{ } \mathrm{Ke}=\sqrt{ } 10^{-14}=10^{-7} \Rightarrow-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log 10^{-7}$

$$
\mathrm{pH}\left(\mathrm{H}_{2} \mathrm{O}\right)=7
$$

## Noticed:

Neutral solution: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{OH}^{-}\right] \Rightarrow \mathrm{pH}=7$.
Acid solution: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]>\left[\mathrm{OH}^{-}\right] \Rightarrow \mathrm{pH}<6.5$.
Basic solution: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]<\left[\mathrm{OH}^{-}\right] \Rightarrow \mathrm{pH}>7.5$.

## $>$ The pH of a strong monoacid (SA):

When a Strong Acid (AH) of initial concentration $\mathrm{C}_{0}$ is dissolved in water, there is total dissociation of the acid:

Couple (HA/A ${ }^{-}$):

|  | AH | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons$ | $\mathrm{A}^{-}$ | $+\mathrm{H}_{3} \mathrm{O}^{+}$ |
| :--- | :--- | :---: | :--- | :--- | :---: |
| $\mathrm{t}=0$ | $\mathrm{C}_{0}$ | - |  | 0 |  |
| $\mathrm{t}=\mathrm{eq}$ | $0=\mathrm{C}_{0}-\mathrm{C}_{0}$ | - |  | $\mathrm{C}_{0}$ |  |
| $\mathrm{C}_{0}$ |  |  |  |  |  |

In the case of slightly diluted solutions (i.e.: $\mathrm{C}_{0}>3.10^{-7} \mathrm{~mol} / \mathrm{l}$ ), the medium is sufficiently acidic so that $\left[\mathrm{OH}^{-}\right]$resulting from the auto-protolysis of water is negligible compared to $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$.

This means that: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{C}_{0} ; \quad \mathrm{pH}=-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log \mathrm{C}_{0}$.
$\Rightarrow \mathrm{pH}=-\log \mathrm{C}_{0} ; \quad\left(\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{C}_{0}\right)$.

## Validity of the approximation:

For me to be sure and the acid is strong it is necessary that: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \geq 10\left[\mathrm{OH}^{-}\right]$.
Demonstration:
$\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \geq 10\left[\mathrm{OH}^{-}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$
$\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \geq 10 \mathrm{Ke} \Rightarrow$ at $\mathrm{t}=25 \mathrm{C}^{\circ}, \mathrm{Ke}=10^{-14} \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \geq 10.10^{-14} \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \geq 10^{-13}$
$\Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \geq \sqrt{ } 10^{-13}=10^{-6.5} \Rightarrow-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \leq-\log 10^{-6.5} \Rightarrow \quad\left(\log 10^{\mathrm{x}}=\mathrm{x}\right)$

$$
\Rightarrow \mathrm{pH} \leq 6.5
$$

## Summary:

Strong Acid (SA):
$\mathbf{p H}(\mathbf{S A})=-\log \mathrm{C}_{\mathbf{0}}\left(\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{C}_{0}\right)$. With $\quad \mathbf{p H} \leq 6.5 \quad\left(\right.$ at $\left.\mathrm{t}=25 \mathrm{C}^{\circ}\right)$

## > The $\mathbf{p H}$ of a strong mono-base ( SB ):

A Strong base (B) with initial concentration $\mathrm{C}_{0}$, We have total protonation of the base:
Couple ( $\mathrm{BH}^{+} / \mathrm{B}$ ):

|  | B | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons$ | $\mathrm{BH}^{+}$ | $+\mathrm{OH}^{-}$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{t}=0$ | $\mathrm{C}_{0}$ | - |  | 0 | 0 |
| $\mathrm{t}=\mathrm{eq}$ | $0=\mathrm{C}_{0}-\mathrm{C}_{0}$ | - |  | $\mathrm{C}_{0}$ |  |
| $\mathrm{C}_{0}$ |  |  |  |  |  |

In the case of slightly diluted solutions (i.e.: $\mathrm{C}_{0}>3.10^{-7} \mathrm{~mol} / \mathrm{l}$ ), the medium is sufficiently basic so that $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$resulting from the auto-protolysis of water is negligible compared to $[\mathrm{OH}$ -]

This means that: $\left[\mathrm{OH}^{-}\right]=\mathrm{C}_{0}$;
$\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]=\mathrm{C}_{0} \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow \mathrm{Ke}=\mathrm{C}_{0} \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{Ke} / \mathrm{C}_{0}$.
$-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log \left(\mathrm{Ke} / \mathrm{C}_{0}\right) \quad(\log \mathrm{A} / \mathrm{B}=\log \mathrm{A}-\log \mathrm{B})$
$\Rightarrow \mathrm{pH}=-\log \mathrm{Ke}+\log \mathrm{C}_{0}, \quad \Rightarrow \mathrm{pH}=\mathrm{pKe}+\log \mathrm{C}_{0}, \quad\left[\mathrm{OH}^{-}\right]=\mathrm{C}_{0}$

## Validity of the approximation:

So that I am sure and the base is strong it is necessary that: $\left[\mathrm{OH}^{-}\right] \geq 10\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$

## Demonstration:

$\left[\mathrm{OH}^{-}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \geq 10\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$. $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$
$\mathrm{Ke} \geq 10\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \Rightarrow$ at $\mathrm{t}=25 \mathrm{C}^{\circ} ; \mathrm{Ke}=10^{-14} \Rightarrow 10^{-14} \geq 10\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \Rightarrow 10^{-15} \geq\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2}$
$\Rightarrow \sqrt{ } 10^{-15}=10^{-7.5} \geq\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow-\log 10^{-7.5} \leq-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \quad\left(\log 10^{\mathrm{x}}=\mathrm{x}\right)$

$$
\Rightarrow \mathrm{pH} \geq 7.5
$$

## Summary:

Strong Base (SB):
$\mathbf{p H}(\mathbf{S B})=\mathbf{p K e}+\log \mathbf{C}_{\mathbf{0}}, \quad\left(\left[\mathrm{OH}^{-}\right]=\mathrm{C}_{0}\right) . \quad$ With $\quad \mathbf{p H} \geq 7.5 \quad\left(\right.$ at $\left.\mathrm{t}=25 \mathrm{C}^{\circ}\right)$

## The $\mathbf{p H}$ of a weak monoacid (WA):

When a weak acid (HA) of initial concentration $\mathrm{C}_{0}$ is dissolved in water, there is a partial dissociation of the acid:

Couple (HA/A $): \quad \mathrm{AH}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{A}^{-}+\mathrm{H}_{3} \mathrm{O}^{+}$(Couple : $\mathrm{HA} / \mathrm{A}^{-}$)

The water auto-protolysis equilibrium: $2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}$
Four species are present at equilibrium concentration: $[\mathrm{AH}],\left[\mathrm{A}^{-}\right],\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$and $\left[\mathrm{OH}^{-}\right]$
We can write four relationships between these four unknowns:

1. Ionic product: $\mathrm{Ke}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$. $\left[\mathrm{OH}^{-}\right]$
2. Couple acidity constant $\left(\mathrm{HA} / \mathrm{A}^{-}\right): \mathrm{Ka}=\left[\mathrm{A}^{-}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /[\mathrm{AH}]$
3. Concentration of the constituent elements of the acid-base couple: $\mathrm{C}_{0}=[\mathrm{AH}]+\left[\mathrm{A}^{-}\right]$
4. Electro-neutrality of the solution: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{A}^{-}\right]+\left[\mathrm{OH}^{-}\right]$

## First approximation:

The medium is acidic: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \gg\left[\mathrm{OH}^{-}\right]$, i.e. the $\left[\mathrm{OH}^{-}\right]$is negligible compared to $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$
The relation (4) becomes: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{A}^{-}\right]$

## Second approximation:

The dissociation of the acid is weak, i.e.: the concentration of the conjugated mono-base is negligible compared to that of the acid AH, this means that: $\left[\mathrm{A}^{-}\right] \ll[\mathrm{AH}]$

The relation (3) becomes: $\mathrm{C}_{0}=[\mathrm{AH}]$
The relation (2) + (5) + (6), we obtain: $\mathrm{Ka}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} /[\mathrm{AH}] \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2}=\mathrm{Ka} \cdot[\mathrm{AH}]$
$\Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\sqrt{ }(\mathrm{Ka} \cdot[\mathrm{AH}]) \Rightarrow-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log (\mathrm{Ka} \cdot[\mathrm{AH}])^{1 / 2} ; \quad(\log \mathrm{A} / \mathrm{B}=\log \mathrm{A}-\log \mathrm{B})$
$\Rightarrow \mathrm{pH}=1 / 2(-\log \mathrm{Ka}-\log [\mathrm{AH}]) \Rightarrow \mathrm{pH}=1 / 2(\mathrm{pKa}-\log [\mathrm{AH}]) \Rightarrow \mathrm{pH}=1 / 2\left(\mathrm{pKa}-\log \mathrm{C}_{0}\right)$.

## Validation Domain:

The medium is acidic: $[\mathrm{AH}] \geq 10\left[\mathrm{~A}^{-}\right]$

## Demonstration:

$[\mathrm{AH}] \geq 10\left[\mathrm{~A}^{-}\right] \Rightarrow 10^{-1} \geq\left[\mathrm{A}^{-}\right] /[\mathrm{AH}] \Rightarrow \log 10^{-1} \geq \log \left[\mathrm{A}^{-}\right] /[\mathrm{AH}] \Rightarrow-1 \geq \log \left[\mathrm{A}^{-}\right] /[\mathrm{AH}]$
$\Rightarrow \mathrm{pKa}-1 \geq \mathrm{pKa}+\log \left[\mathrm{A}^{-}\right] /[\mathrm{AH}]$
According to Henderson relation: $\mathrm{pH}=\mathrm{pKa}+\log$ ([base]/[acid])
$\Rightarrow \mathrm{pKa}-1 \geq \mathrm{pH} \quad \Rightarrow \quad \mathrm{pH} \leq \mathrm{pKa}-1$.

## Use of the dissociation coefficient $\left(\alpha_{a}\right)$ :

The weak acid approximation relies on the assumption of weak dissociation.
We call: $\left(\alpha_{a}\right)$ : dissociation coefficient $=$ Number of moles dissociated from AH/Initial number of moles of AH, $\quad \alpha_{a}=x / C_{0}$

## Material equilibruim:

|  | AH | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons$ | $A^{-}$ | $+\mathrm{H}_{3} O^{+}$ |
| :--- | :--- | :---: | :--- | :---: | :---: |
| $\mathrm{t}=0$ | $\mathrm{C}_{0}$ | - |  | 0 | 0 |
| $\mathrm{t}=\mathrm{eq}$ | $\mathrm{C}_{0}-\mathrm{X}$ | - |  | X |  |
|  |  |  |  |  |  |
| $\alpha_{\mathrm{a}}=\mathrm{X} / \mathrm{C}_{0}$ | $\mathrm{C}_{0}\left(1-\alpha_{\mathrm{a}}\right)$ | - |  | $\mathrm{C}_{0} \alpha_{\mathrm{a}}$ | X |
|  |  |  | $\mathrm{C}_{0} \alpha_{\mathrm{a}}$ |  |  |

$$
\mathrm{Ka}=\left[\mathrm{A}^{-}\right] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /[\mathrm{AH}]=\mathrm{C}_{0} \alpha_{\mathrm{a}} \cdot \mathrm{C}_{0} \alpha_{\mathrm{a}} / \mathrm{C}_{0}\left(1-\alpha_{\mathrm{a}}\right)=\mathrm{C}_{0} \alpha_{\mathrm{a}}^{2} /\left(1-\alpha_{\mathrm{a}}\right) \Rightarrow \mathrm{Ka}=\mathrm{C}_{0} \alpha_{\mathrm{a}}^{2} /\left(1-\alpha_{\mathrm{a}}\right)
$$

- If $\alpha_{a} \leq 0.1$ (the dissociation is weak; less than $\left.10 \%\right) \Rightarrow$ we can neglect the $\alpha_{a}$ in front of 1 :

The relationship becomes: $\mathrm{Ka}=\mathrm{C}_{0} . \alpha_{\mathrm{a}}{ }^{2} \Rightarrow \alpha_{\mathrm{a}}=\sqrt{ }\left(\mathrm{ka} / \mathrm{C}_{0}\right) \leq 10^{-1} \Rightarrow \mathrm{ka} / \mathrm{C}_{0} \leq 10^{-2}$.

## Note:

This is Ostwald's law which indicates that dissociation increases with dilution and a much diluted weak electrolyte behaves like a strong electrolyte. This is why the calculation of the pH of acids and bases is applicable for solutions which are not too diluted.

## Summary:

Weak acid (WA):
$\mathrm{pH}(\mathrm{WA})=1 / 2\left(\mathrm{pKa}-\log \mathrm{C}_{0}\right) \quad . \quad\left(\mathrm{t}=25 \mathrm{C}^{\circ}\right)$
With: 1/ $\mathrm{pH} \leq \mathrm{pKa}-1$; 2/If $\alpha_{\mathrm{a}} \leq 0.1 \Rightarrow \mathrm{Ka} / \mathrm{C}_{0} \leq 10^{-2}$.

## $>$ The pH of a weak mono-base (WB):

A weak base (B) with initial concentration $\mathrm{C}_{0}$, We have partial protonation of the base:

Couple ( $\left.\mathrm{BH}^{+} / \mathrm{B}\right): \quad \mathrm{B}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{BH}^{+}+\mathrm{OH}^{-}$
The water auto-protolysis equilibrium: $2 \mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-}$
Four species are present at equilibrium concentration: $[\mathrm{B}],\left[\mathrm{BH}^{+}\right],\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$and $\left[\mathrm{OH}^{-}\right]$
We can write four relationships between these four unknowns:

1. Ionic product: $\mathrm{Ke}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]$
2. Couple basicity constant $\left(\mathrm{BH}^{+} / \mathrm{B}\right): \mathrm{Kb}=\left[\mathrm{BH}^{+}\right]$. $\left[\mathrm{OH}^{-}\right] /[\mathrm{B}]$ $\qquad$
3. Concentration of the constituent elements of the acid-base couple: $\mathrm{C}_{0}=[\mathrm{B}]+\left[\mathrm{BH}^{+}\right]$
4. Electro-neutrality of the solution: $\left[\mathrm{BH}^{+}\right]+\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{OH}^{-}\right]$

## First approximation:

The medium is basic: $\left[\mathrm{OH}^{-}\right] \gg\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$, i.e. the $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$is negligible compared to $\left[\mathrm{OH}^{-}\right]$
The relation (4) becomes: $\left[\mathrm{BH}^{+}\right]=\left[\mathrm{OH}^{-}\right]$

## Second approximation:

The protonation of the base is weak, i.e.: the concentration of the conjugated monoacid is negligible compared to that of base B, this means that: $\left[\mathrm{BH}^{+}\right] \ll[B]$.

The relation (3) becomes: $\mathrm{C}_{0}=[\mathrm{B}]$
$\mathrm{Kb}=\left[\mathrm{BH}^{+}\right] .\left[\mathrm{OH}^{-}\right] /[\mathrm{B}] \ldots .$. (2) $\quad\left(\mathrm{Ka} \cdot \mathrm{Kb}=\mathrm{Ke} ; \mathrm{Ke}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]\right)$
$\Rightarrow \mathrm{Ke} / \mathrm{Ka}=\left[\mathrm{BH}^{+}\right] \mathrm{Ke} /[\mathrm{B}]\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow \mathrm{Ke} \cdot[\mathrm{B}] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{Ka} \cdot\left[\mathrm{BH}^{+}\right] \cdot \mathrm{Ke}$
$\Rightarrow \mathrm{Ka}=[\mathrm{B}] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{BH}^{+}\right]$
(7) $+(5)+(6)$, we obtained: $\mathrm{Ka}=\mathrm{C}_{0} .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{OH}^{-}\right] \Rightarrow \mathrm{Ka}=\mathrm{C}_{0} .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} / \mathrm{Ke}$
$\Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2}=\left(\mathrm{Ka} \mathrm{Ke} / \mathrm{C}_{0}\right) \Rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\sqrt{ }\left(\mathrm{Ka} \mathrm{Ke} / \mathrm{C}_{0}\right) \Rightarrow-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log \left(\mathrm{Ke} \mathrm{Ka} / \mathrm{C}_{0}\right)^{1 / 2}$;
$(\log \mathrm{A} / \mathrm{B}=\log \mathrm{A}-\log \mathrm{B}, \log \mathrm{A} \cdot \mathrm{B}=\log \mathrm{A}+\log \mathrm{B})$
$\Rightarrow \mathrm{pH}=1 / 2\left(-\log \mathrm{Ka}-\log \mathrm{Ka}+\log \mathrm{C}_{0}\right) \Rightarrow \mathrm{pH}=1 / 2\left(\mathrm{pKe}+\mathrm{pKa}+\log \mathrm{C}_{0}\right)$

## Validation Domain:

The medium is basic: $[B] \geq 10\left[\mathrm{BH}^{+}\right]$

## Demonstration:

$[\mathrm{B}] \geq 10\left[\mathrm{BH}^{+}\right] \Rightarrow 10^{-1} \geq\left[\mathrm{BH}^{+}\right] /[\mathrm{B}] \Rightarrow[\mathrm{B}] /\left[\mathrm{BH}^{+}\right] \geq 10 \Rightarrow \log [\mathrm{~B}] /\left[\mathrm{BH}^{+}\right] \geq \log 10$
$\Rightarrow \mathrm{pKa}+\log [\mathrm{B}] /\left[\mathrm{BH}^{+}\right] \geq \mathrm{pKa}+1$ (Henderson relation: $\mathrm{pH}=\mathrm{pKa}+\log$ ([base]/[acid] $)$
$\Rightarrow \mathrm{pH} \geq \mathrm{pKa}+1$.

## Use of the dissociation coefficient ( $\alpha_{b}$ ):

The weak basis approximation relies on the assumption of weak protonation
We call: $\left(\alpha_{b}\right)$ : dissociation coefficient $=$ Number of moles dissociated from $[B] /$ Initial number of moles of $[B], \alpha_{b}=x / C_{0}$.

## Material equilibruim:

|  | $B$ | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons$ | $\mathrm{BH}^{+}$ | $+\mathrm{OH}^{-}$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{t}=0$ | $\mathrm{C}_{0}$ | - |  | 0 |  |
| $\mathrm{t}=\mathrm{eq}$ | $\mathrm{C}_{0}-\mathrm{X}$ | - |  | X |  |
| $\alpha_{\mathrm{b}}=\mathrm{X} / \mathrm{C}_{0}$ | $\mathrm{C}_{0}\left(1-\alpha_{\mathrm{b}}\right)$ |  | - |  | $\mathrm{C}_{0} \alpha_{\mathrm{b}}$ |
| X |  |  |  |  |  |

$\mathrm{Kb}=\left[\mathrm{BH}^{+}\right] .\left[\mathrm{OH}^{-}\right] /[\mathrm{B}] \quad\left(\mathrm{Ka} \cdot \mathrm{Kb}=\mathrm{Ke} ; \mathrm{Ke}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] .\left[\mathrm{OH}^{-}\right]\right)$
$\Rightarrow \mathrm{Ke} / \mathrm{Ka}=\left[\mathrm{BH}^{+}\right] \mathrm{Ke} /[\mathrm{B}]\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow \mathrm{Ke} \cdot[\mathrm{B}] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\mathrm{Ka} \cdot\left[\mathrm{BH}^{+}\right] . \mathrm{Ke}$
$\Rightarrow \mathrm{Ka}=[\mathrm{B}] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{BH}^{+}\right] \Rightarrow \mathrm{Ka}=\mathrm{Ke} \cdot[\mathrm{B}] /\left[\mathrm{BH}^{+}\right]\left[\mathrm{OH}^{-}\right] \Rightarrow \mathrm{Ka}=\mathrm{Ke} \mathrm{C}_{0}\left(1-\alpha_{\mathrm{b}}\right) / \mathrm{C}_{0} \alpha_{\mathrm{b}} \cdot \mathrm{C}_{0} \alpha_{\mathrm{b}}$
$\Rightarrow \mathrm{Ka}=\mathrm{Ke}\left(1-\alpha_{\mathrm{b}}\right) / \mathrm{C}_{0} \alpha_{\mathrm{b}}{ }^{2}$

- If $\alpha_{b} \leq 0.1$ (the protonation is weak; less than $\left.10 \%\right) \Rightarrow$ we can neglect the $\alpha_{b}$ in front of 1 :

The relationship becomes: $\mathrm{Ka}=\mathrm{Ke} / \mathrm{C}_{0} \alpha_{\mathrm{b}}{ }^{2} \Rightarrow \alpha_{\mathrm{b}}=\sqrt{ }\left(\mathrm{Ke} / \mathrm{KaC}_{0}\right) \leq 10^{-1} \Rightarrow \mathrm{Ke} / \mathrm{KaC}_{0} \leq 10^{-2}$.

## Summary:

Weak base (WB):
$\mathrm{pH}(\mathrm{WB})=1 / 2\left(\mathrm{pKe}+\mathrm{pKa}+\log \mathrm{C}_{0}\right) \quad . \quad\left(\mathrm{t}=25 \mathrm{C}^{\circ}\right)$
With: $\mathbf{1} / \mathrm{pH} \geq \mathrm{pKa}+1 ; 2 /$ If $\alpha_{\mathrm{b}} \leq 0.1 \Rightarrow \mathrm{Ke} / \mathrm{KaC}_{0} \leq 10^{-2}$.

## The $\mathbf{p H}$ of an amphoteric solution:

Consider the solution of a NaHA salt (Example: $\mathrm{Na} \mathrm{HCO}_{3}$ ) of concentration C.
The total dissociation of salt in water is written as: $\mathrm{NaHA}_{\text {(solid) }} \rightarrow \mathrm{Na}^{+}+\mathrm{HA}^{-}$
$\mathrm{HA}^{-}$is an ampholyte since it is the acid pair: $\mathrm{HA}^{-} / \mathrm{A}^{-2}$ and the base of $\mathrm{H}_{2} \mathrm{~A} / \mathrm{HA}^{-}$

| Couple $\left(\mathrm{H}_{2} \mathrm{~A}^{2} / \mathrm{HA}^{-}\right):$ | $\mathrm{H}_{2} \mathrm{~A}$ | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons \mathrm{HA}^{-}$ | $+\mathrm{H}_{3} \mathrm{O}^{+}$ |
| :--- | :--- | :--- | :--- | :--- |
| Couple $\left(\mathrm{HA}^{-} / \mathrm{A}^{-2}\right):$ | $\mathrm{HA}^{-}$ | $+\mathrm{H}_{2} \mathrm{O}$ | $\rightleftharpoons \mathrm{A}^{-2}$ | $+\mathrm{H}_{3} \mathrm{O}^{+}$ |
| Totale : | $2 \mathrm{HA}^{-}$ |  | $\rightleftharpoons \mathrm{H}_{2} \mathrm{~A}$ | $+\mathrm{A}^{-2}$ |

The balance of this overall reaction indicates that: $\left[\mathrm{H}_{2} \mathrm{~A}\right]=\left[\mathrm{A}^{-2}\right]$
Ka 1 : acidity constant: $\mathrm{H}_{2} \mathrm{~A} / \mathrm{HA}^{-} \Rightarrow \mathrm{Ka}_{1}=\left[\mathrm{HA}^{-}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{H}_{2} \mathrm{~A}\right]$
Ka 2 : acidity constant: $\mathrm{HA}^{-} / \mathrm{A}^{-2} \Rightarrow \mathrm{Ka}_{2}=\left[\mathrm{A}^{-2}\right] .\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{HA}^{-}\right]$
$\mathrm{Ka}_{1} \cdot \mathrm{Ka}_{2}=\left(\left[\mathrm{HA}^{-}\right] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{H}_{2} \mathrm{~A}\right]\right) \cdot\left(\left[\mathrm{A}^{-2}\right] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] /\left[\mathrm{HA}^{-}\right]\right)=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \cdot\left(\left[\mathrm{~A}^{-2}\right] /\left[\mathrm{H}_{2} \mathrm{~A}\right]\right.$
$\left[\mathrm{H}_{2} \mathrm{~A}\right]=\left[\mathrm{A}^{-2}\right]$
$\mathrm{Ka}_{1} \cdot \mathrm{Ka}_{2}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]^{2} \Rightarrow \mathrm{pH}=1 / 2(\mathrm{pKa} 1+\mathrm{pKa} 2)$ (We note that pH is independent of concentration (C)).

## Noticed:

An ampholyte is a substance that contains a negative charge and hydrogen (regardless of the number of negative charges or hydrogen atoms).

