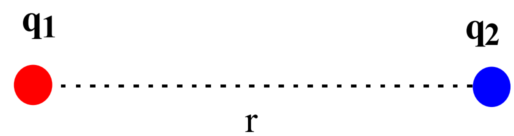
Electrostatic Force and Electric Charge

Electrostatic Force (charges at rest):



- **Electrostatic force** can be **attractive**
- **Electrostatic force** can be **repulsive**
- **Electrostatic force** acts through **empty space**
- **Electrostatic force** much stronger than **gravity**
- **Electrostatic forces** are **inverse square law** forces (**proportional to $1/r^2$**)
- **Electrostatic force** is proportional to the product of the amount of charge on each interacting object

Magnitude of the Electrostatic Force is given by Coulomb's Law:

$$F = K q_1 q_2/r^2 \qquad \textbf{(Coulomb's Law)}$$

where K depends on the system of units

$$K = 8.99\times10^{9}\ Nm^2/C^2 \quad \text{(in MKS system)}$$
$$K = 1/(4\pi\varepsilon_0) \quad \text{where} \quad \varepsilon_0 = 8.85\times10^{-12}\ C^2/(Nm^2)$$

Electric Charge:

electron charge = -e $\qquad$ **$e = 1.6\times10^{-19}$ C**
proton charge = e $\qquad$ C = Coulomb

Electric charge is a conserved quantity (*net electric charge is never created or destroyed!*)

Units

MKS System (meters-kilograms-seconds):

also Amperes, Volts, Ohms, Watts

Force:	$F = ma$	Newtons = kg m / s^2 = 1 N
Work:	$W = Fd$	Joule = Nm = kg m^2 / s^2 = 1 J
Electric Charge:	Q	Coulomb = 1 C
$F = K\, q_1 q_2/r^2$	$K = 8.99\times 10^9\ Nm^2/C^2$	(in MKS system)

CGS System (centimeter-grams-seconds):

Force:	$F = ma$	1 dyne = g cm / s^2
Work:	$W = Fd$	1 erg = dyne-cm = g cm^2 / s^2
Electric Charge:	Q	esu (electrostatic unit)
$F = q_1 q_2/r^2$	$K = 1$	(in CGS system)

Conversions (MKS - CGS):

Force:	1 N = 10^5 dynes
Work:	1 J = 10^7 ergs
Electric Charge:	1 C = 2.99×10^9 esu

Fine Structure Constant (dimensionless):

$$\alpha = K\, 2\pi e^2/hc$$

(same in all systems of units)

h = Plank's Constant c = speed of light in vacuum

Electrostatic Force versus Gravity

Electrostatic Force :

$F_e = K\, q_1 q_2 / r^2$ **(Coulomb's Law)**

$K = 8.99\text{x}10^{9}\ Nm^2/C^2$ **(in MKS system)**

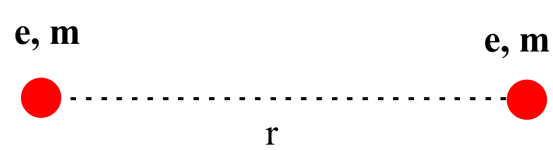
Gravitational Force :

$F_g = G\, m_1 m_2 / r^2$ **(Newton's Law)**

$G = 6.67\text{x}10^{-11}\ Nm^2/kg^2$ **(in MKS system)**

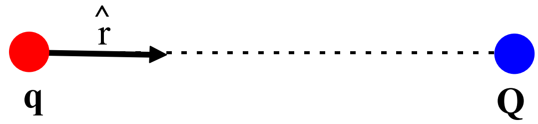
Ratio of forces for two electrons :

$e = 1.6\text{x}10^{-19}\ C$ $m = 9.11\text{x}10^{-31}\ kg$



$F_e / F_g = K e^2 / G m^2 = 4.16\text{x}10^{42}$ **(Huge number !!!)**

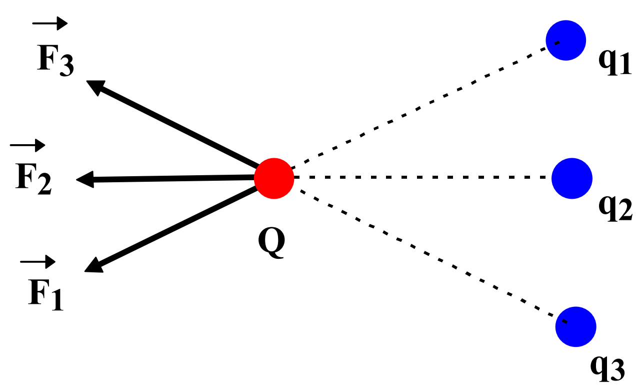
Vector Forces



The Electrostatic Force is a vector:

The force on q due to Q points along the direction r and is given by

$$\vec{F} = \frac{KqQ}{r^2}\hat{r}$$



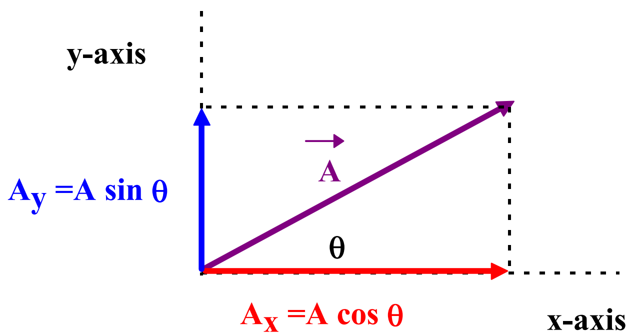
Vector Superposition of Electric Forces:

If several point charges q_1, q_2, q_3, … simultaneously exert electric forces on a charge Q then

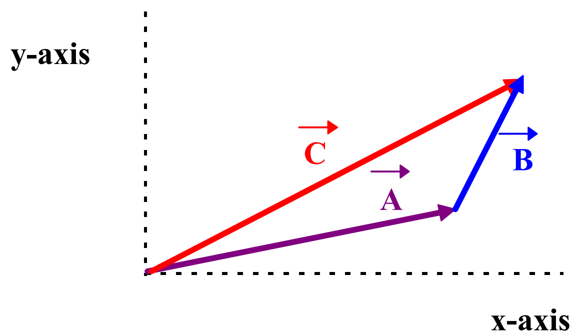
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

Vectors & Vector Addition

The Components of a vector:



Vector Addition:



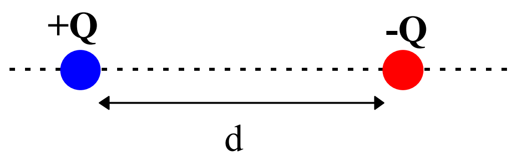
To add vectors you add the components of the vectors as follows:

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

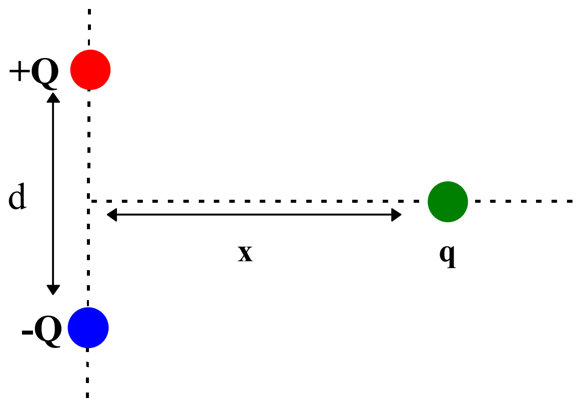
$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

The Electric Dipole



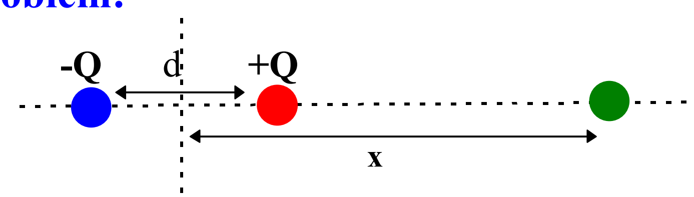
An electric "dipole" is two equal and opposite point charges separated by a distance d. It is an electrically neutral system. The "dipole moment" is defined to be the charge times the separation (dipole moment = Qd).

Example Problem:



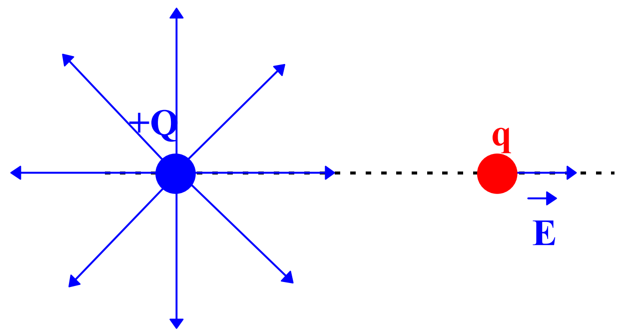
A dipole with charge **Q** and separation **d** is located on the y-axis with its midpoint at the origin. A charge **q** is on the x-axis a distance **x** from the midpoint of the dipole. What is the electric force on **q** due to the dipole and how does this force behave in the limit **x >>d** (**dipole approximation**)?

Example Problem:



A dipole with charge **Q** and separation **d** is located on the x-axis with its midpoint at the origin. A charge **q** is on the x-axis a distance **x** from the midpoint of the dipole. What is the electric force on **q** due to the dipole and how does this force behave in the limit **x >>d** (**dipole approximation**)?

The Electric Field



The charge Q produces an electric field which in turn produces a force on the charge q. The force on q is expressed as two terms:

$$F = K\,qQ/r^2 = q\,(KQ/r^2) = q\,E$$

The electric field at the point q due to Q is simply the force per unit positive charge at the point q:

$$E = F/q \qquad\qquad E = KQ/r^2$$

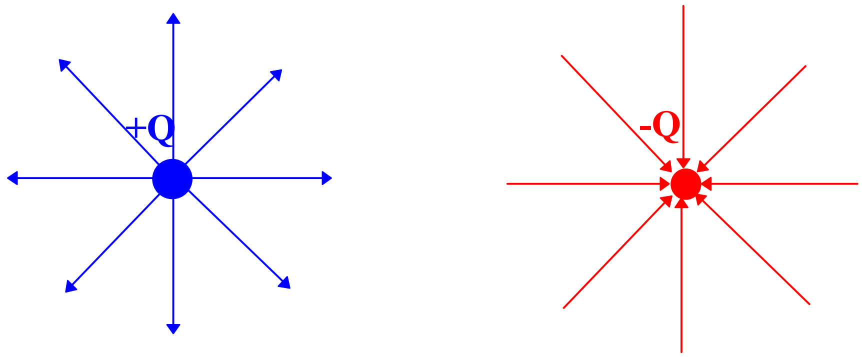
The units of E are Newtons per Coulomb (units = N/C).

The electric field is a physical object which can carry both momentum and energy. It is the mediator (or carrier) of the electric force. The electric field is massless.

The Electric Field is a Vector Field:

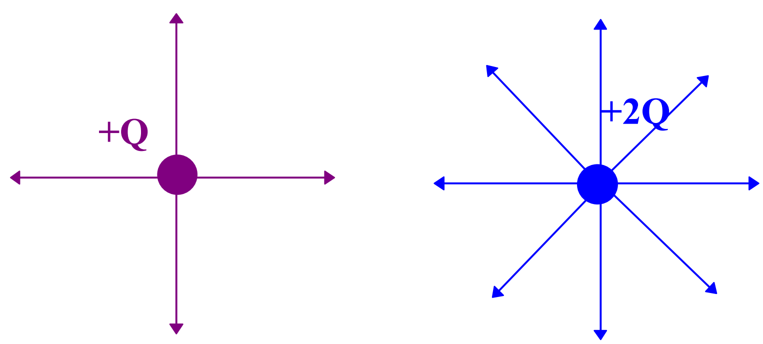
$$\vec{E} = \frac{KQ}{r^2}\hat{r}$$

Electric Field Lines

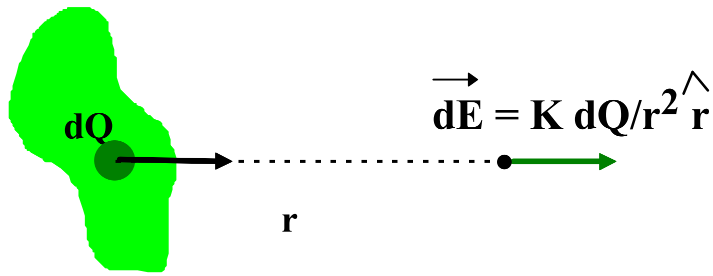


Electric field line diverge from (i.e. start) on positive charge and end on negative charge. The direction of the line is the direction of the electric field.

The number of lines penetrating a unit area that is perpendicular to the line represents the strength of the electric field.



Electric Field due to a Distribution of Charge

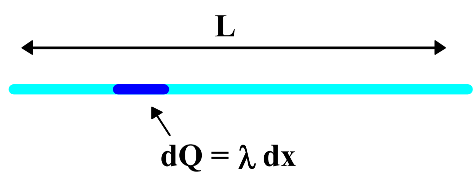


The electric field from a continuous distribution of charge is the superposition (i.e. integral) of all the (infinite) contributions from each infinitesimal dQ as follows:

$$\vec{E} = \int \frac{K}{r^2}\hat{r}\,dQ \qquad \text{and} \qquad Q = \int dQ$$

Charge Distributions:

- **Linear charge density λ:** **λ(x) = charge/unit length**



For a straight line dQ = λ(x) dx and

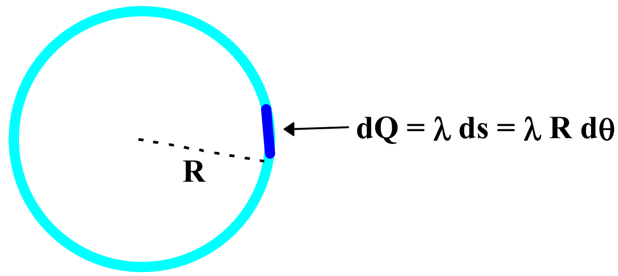
$$Q = \int dQ = \int \lambda(x)dx$$

If λ(x) = λ is constant then dQ = λ dx and Q = λL, where L is the length.

Charge Distributions

Charge Distributions:

- **Linear charge density λ:** **λ(θ) = charge/unit arc length**

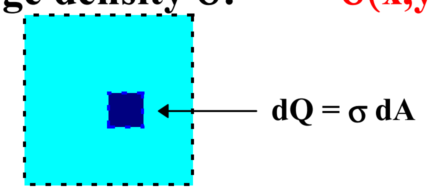


For a circular arc dQ = λ(θ) ds = λ(θ) Rdθ and

$$Q = \int dQ = \int \lambda(\theta) ds = \int \lambda(\theta) R d\theta$$

If λ(θ) = λ is constant then dQ = λ ds and Q = λs, where s is the arc length.

- **Surface charge density σ:** **σ(x,y) = charge/unit area**

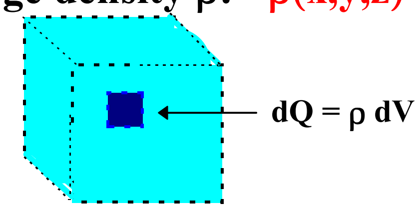


For a surface dQ = σ(x,y) dA and

$$Q = \int dQ = \int \sigma(x,y) dA$$

If σ(x,y) = σ is constant then dQ = σ dA and Q = σA, where A is the area.

- **Volume charge density ρ:** **ρ(x,y,z) = charge/unit volume**



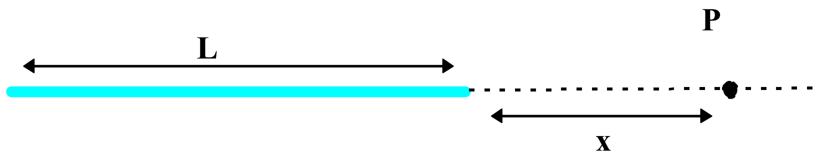
For a surface dQ = ρ(x,y,z) dV and

$$Q = \int dQ = \int \rho(x,y,z) dV$$

If ρ(x,y,z) = ρ is constant then dQ = ρ dV and Q = ρV, where V is the volume.

Calculating the Electric Field

Example:

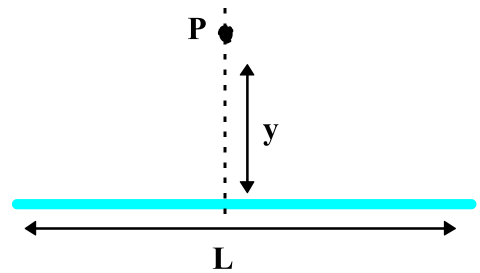


A total amount of charge **Q** is uniformly distributed along a thin **straight rod** of length **L**. What is the electric field a a point **P** on the x-axis a distance **x** from the end of the rod?

Answer: $$\vec{E} = \frac{KQ}{x(x+L)}\hat{x}$$

Example:

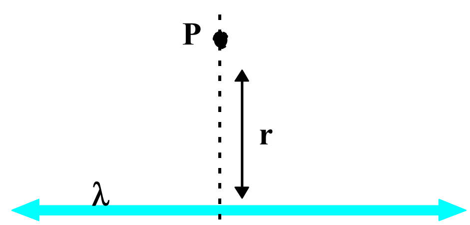
A total amount of charge **Q** is uniformly distributed along a thin **straight rod** of length **L**. What is the electric field a a point **P** on the y-axis a distance **y** from the midpoint of the rod?



Answer: $$\vec{E} = \frac{KQ}{y\sqrt{y^2+(L/2)^2}}\hat{y}$$

Example:

A **infinitely long straight rod** has a uniform charge density λ. What is the electric field a point **P** a perpendicular distance r from the rod?



Answer: $$\vec{E} = \frac{2K\lambda}{r}\hat{r}$$

Some Useful Math

Approximations:

$$(1+\varepsilon)^p \underset{\varepsilon<<1}{\approx} 1+p\varepsilon$$

$$(1-\varepsilon)^p \underset{\varepsilon<<1}{\approx} 1-p\varepsilon$$

$$e^{\varepsilon} \underset{\varepsilon<<1}{\approx} 1+\varepsilon$$

$$\tan\varepsilon \underset{\varepsilon<<1}{\approx} \varepsilon \qquad \sin\varepsilon \underset{\varepsilon<<1}{\approx} \varepsilon$$

Indefinite Integrals:

$$\int \frac{a^2}{\left(x^2+a^2\right)^{3/2}}dx = \frac{x}{\sqrt{x^2+a^2}}$$

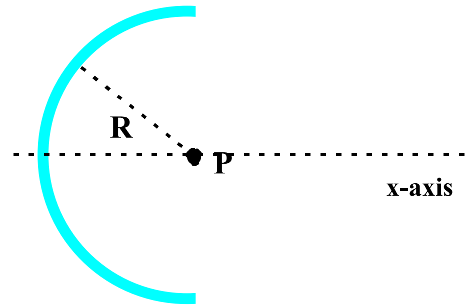
$$\int \frac{x}{\left(x^2+a^2\right)^{3/2}}dx = \frac{-1}{\sqrt{x^2+a^2}}$$

Calculating the Electric Field

Example:

A total amount of charge **Q** is uniformily distributed along a thin **semicircle** of radius **R**. What is the electric field a a point **P** at the center of the circle?

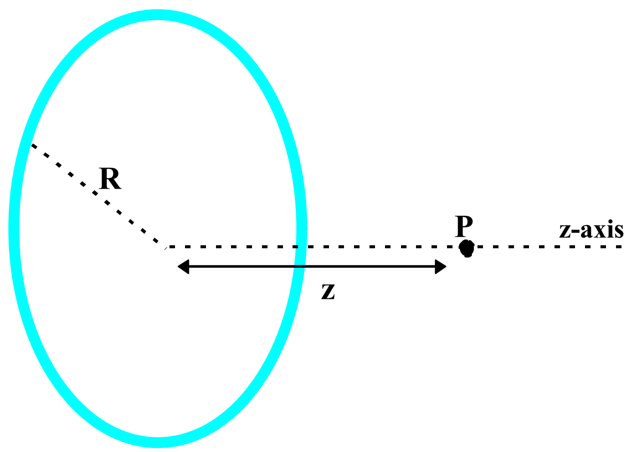
Answer: $$\vec{E} = \frac{2KQ}{\pi R^2}\hat{x}$$



Example:

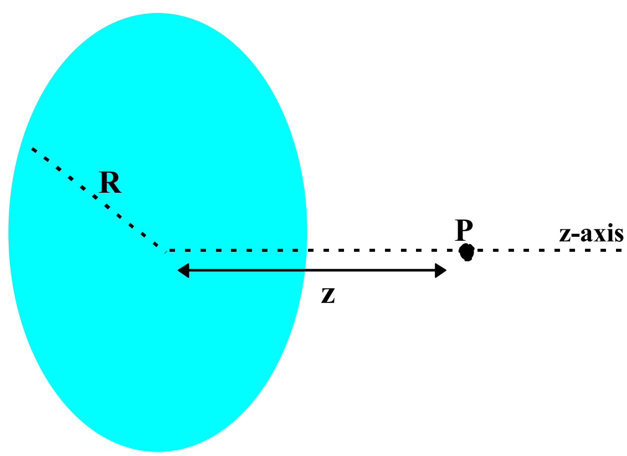
A total amount of charge **Q** is uniformily distributed along a thin **ring** of radius **R**. What is the electric field a point **P** on the z-axis a distance **z** from the center of the ring?

Answer: $$\vec{E} = \frac{KQz}{(z^2+R^2)^{3/2}}\hat{z}$$



Example:

A total amount of charge **Q** is uniformily distributed on the surface of a **disk** of radius **R**. What is the electric field a point **P** on the z-axis a distance **z** from the center of the disk?



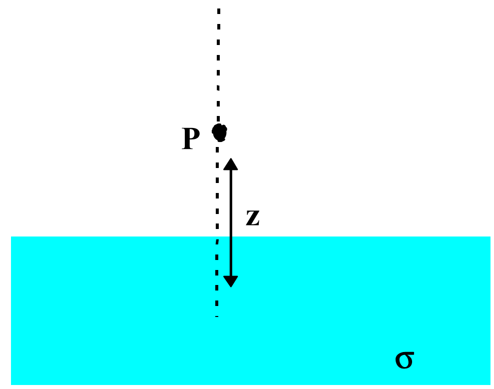
Answer:

$$\vec{E} = \frac{2KQ}{R^2}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}$$

Calculating the Electric Field

Example:

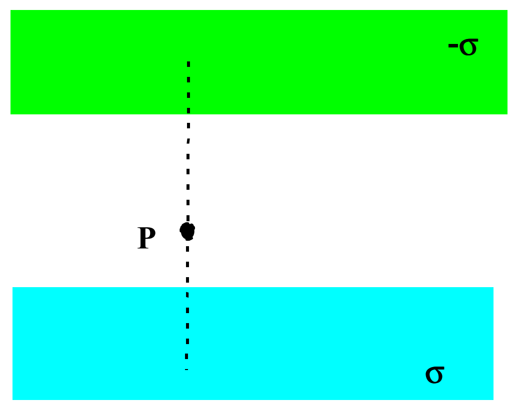
What is the electric field generated by a large (**infinite**) **sheet** carrying a uniform surface charge density of **σ** coulombs per meter?



Answer: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$

Example:

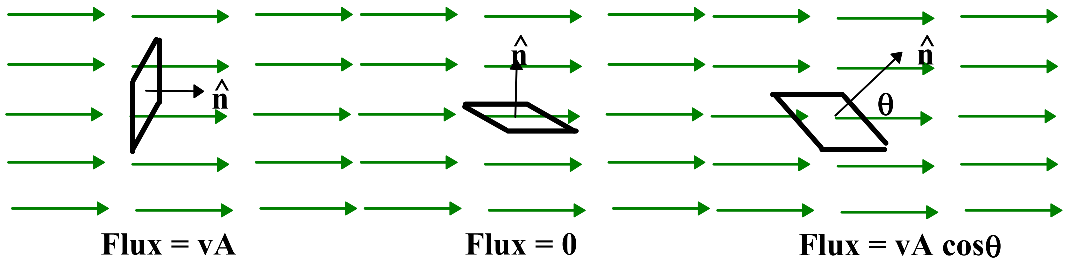
What is the electric field at a point P between **two** large (**infinite**) **sheets** carrying an equal but opposite uniform surface charge density of **σ**?



Answer: $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}$

Flux of a Vector Field

Fluid Flow:



Consider the fluid with a vector $\vec{v}$ which describes the velocity of the fluid at every point in space and a square with area A = L^2 and normal $\hat{n}$. **The flux is the volume of fluid passing through the square area per unit time.**

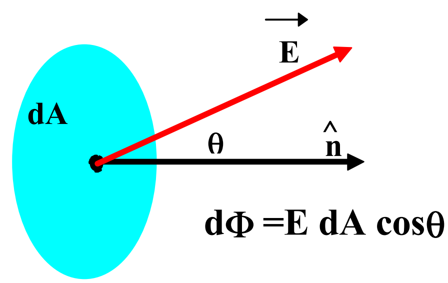
Generalize to the Electric Field:

Electric flux through the infinitesimal area dA is equal to

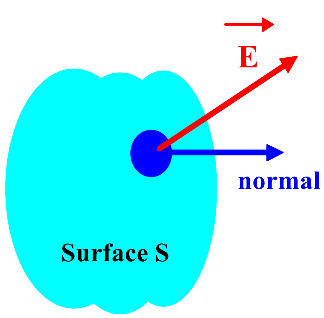
$$d\Phi = \vec{E} \cdot d\vec{A}$$

where

$$d\vec{A} = A\hat{n}$$



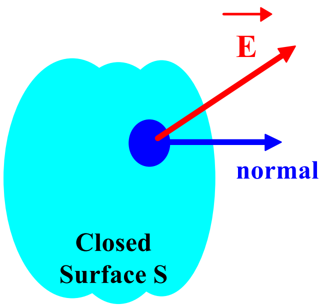
Total Electric Flux through a Closed Surface:



$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$

Electric Flux and Gauss' Law

The electric flux through any closed surface is proportional to the net charge enclosed.



$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

For the discrete case the total charge enclosed is the sum over all the enclosed charges:

$$Q_{enclosed} = \sum_{i=1}^{N} q_i$$

For the continuous case the total charge enclosed is the integral of the charge density over the volume enclosed by the surface S:

$$Q_{enclosed} = \int \rho\, dV$$

Simple Case: If the electric field is constant over the surface and if it always points in the same direction as the normal to the surface then

$$\Phi_E = \oint_S \vec{E}\cdot d\vec{A} = EA$$

The units for the electric flux are **Nm^2/C**.