University of Biskra
Mathematics Department
Module: Analysis 2

First year license
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Worksheet N°0

Exercise 1 Calculate the derivatives of the following univariate functions:

1. $f(x,y) = e^{x^2+y}$, $\quad g(x,y) = x^2 + y^2$, $\quad h(x,y) = x^y$.
2. $\arcsin\left(\frac{x}{\sqrt{a}}\right)$, $\quad \arccos\left(\frac{x}{\sqrt{a}}\right)$, $\quad \arctan\left(\frac{x}{a}\right)$, with $a > 0$.

Exercise 2 Let's consider the following functions:

$$\begin{aligned}
f_1(x) &= ln\left(\frac{\alpha x + \beta}{ax + b}\right).\\
f_2(x) &= \frac{\alpha}{2a}\ \ln(ax^2 + b) + \frac{\beta}{\sqrt{ab}}\ \arctan\left(\sqrt{\frac{a}{b}}\ x\right), \text{ with } a, b > 0.\\
f_3(x) &= \frac{\alpha}{\sqrt{a}}\ \arcsin\left(\sqrt{\frac{a}{b}}\ x\right) - \frac{\beta}{a}\ \sqrt{b - ax^2}, \text{ with } a, b > 0.\\
f_4(x) &= \sum_{k=0}^{q} \frac{(-1)^k\ C_q^k}{(n + 2k + 1)} \cos(x)^{n+2k+1}, \qquad \text{with } n, k, q \in \mathbb{N} \text{ and } C_q^k = \frac{q!}{(q-k)!\ k!}.
\end{aligned}$$

1. Give the derivatives of the functions f_1, f_2 and f_3 in their simplified form.
2. Check that f_4' can be expressed as a product of $\cos^n$ and $\sin^m$ (n and m are two natural numbers).
3. Check that the proposition of the second question remains true when we replace *cos* with *sin* in the expression of f_4.

Exercise 3 Considering the polynomial $P_2(x) = x^2 + ax + b$, with $a, b \in \mathbb{R}$. Show that:

1. If $a^2 - 4b \geq 0$ then $P_2(x)$ can be rewritten $P_2(x) = (x + A)^2 - B^2$.
2. If $a^2 - 4b < 0$ then $P_2(x)$ can be rewritten $P_2(x) = (x + A)^2 + B^2$.

Exercise 4 Determine the expression of the function f that must be differentiated to obtain the following:

$$\begin{aligned}
f'(x) &= \cos(x), & f'(x) &= x + 2x^{\sqrt{2}} + 1, & f'(x) &= \tfrac{x+1}{1+x^2},\\
f'(x) &= \tfrac{x+1}{\sqrt{1-x^2}}, & f'(x) &= e^{\frac{x}{2}} + 1
\end{aligned}$$