University of Biskra
Mathematics Department
Module: Analysis 1

First year license
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Exercise series-course

Exercise 1 The plan is related to an orthonormal reference frame. Let f be the real variable function defined by

$$f(x) = \frac{x^2 - x - 2}{x^2 - x + 1}.$$

Let $\mathcal{C}_f$ its graphic representation in an orthonormal reference frame.

1. What is the domain D_f of f?
2. What is the differentiability set of f?
3. Calculate the derivative f' of the function f.
4. Determine the sign of f', then deduce the variations direction of f.
5. Determine the limits of f at the bounds of D_f.
6. Determine possible asymptotes and their relative position to $\mathcal{C}_f$.
7. Determine the equation of the tangent T to $\mathcal{C}_f$ at its point of abscissa 2.
8. Study the relative position of $\mathcal{C}_f$ and T.
9. Construct, on the same drawing, the curve $\mathcal{C}_f$, the tangent T and the asymptotes.

Exercise 2 The plan is related to an orthonormal reference frame. Let f be the real function defined by

$$f(x) = \frac{x^3 - 4x^2 + 8x - 4}{(x-1)^2}$$

. Let $\mathcal{C}_f$ its graphic representation in an orthonormal reference frame.

1. What is the domain D_f of f?
2. Determine three real numbers a, b and c such that for all $x \in D_f$:

$$f(x) = x + a + \frac{b}{x-1} + \frac{c}{(x-1)^2}.$$

3. Specify the position of $\mathcal{C}_f$ to the line D with the equation $y = x - 2$.
4. Determine the differentiability set of f. Calculate the derivative f' of the function f.
5. Determine the sign of f', then deduce the variations direction of f.
6. Determine the limits of f across D_f.
7. Determine possible asymptotes and their relative position to $\mathcal{C}_f$.
8. Determine the equation of the tangent T to $\mathcal{C}_f$ at its point of abscis $x = 2$.
9. Construct, on the same frame, the curve $\mathcal{C}_f$, the line D as well as the line with equation $x = 1$ and the tangent T.

Exercise 3

1. Find the first 3 terms of the Taylor series for the function $sin(\pi x)$ centered at $a = 0.5$. Then, use your answer to find an approximate value to $sin(\frac{\pi}{2} + \frac{\pi}{10})$

2. Find the first 4 terms in the Taylor series for $(x-1)e^x$ near $x = 1$.

3. Find the Maclaurin series for $x\ sinx$.

4. Find the Maclaurin series for $ln\left(\frac{1+x}{1-x}\right)$.

5. (a) Find the Maclaurin series for $(1+x)^m$ where m is not necessarily an integer and hence show that the formula for the binomial series works for non-integral exponents as well. (b) Use your answer to find the expansion of $\frac{1}{\sqrt{1-x^2}}$ up to the term in x^6.

6. Find the first 3 terms in the Maclaurin series for $cos(sinx)$. Hence or otherwise find

$$\lim_{x\to 0} \frac{1 - cos(sin(x))}{x^2}.$$

7. Using the Taylor series, determine the following limits:

1) $\lim_{x\to 0} \frac{e^{3x-2}-e^2}{x}$ 2) $\lim_{x\to 1} \frac{ln(2-x)}{x-1}$ 3) $\lim_{x\to \pi} \frac{sin(x)}{x^2-\pi^2}$ 4) $\lim_{x\to \frac{\pi}{2}} \frac{e^{cos(x)}}{x-\frac{\pi}{2}}$ 5) $\lim_{x\to 0} \frac{ln(1-sin(x))}{x}$