الجمهورية الجزائرية الديمقراطية الشعبية

وزارة التعليم العالي والبحث العلمي

MINISTERE DE L' ENSEIGNEMENT SUPERIEUR ET DE LA RECHERCHE SCIENTIFIQUE

Direction Générale de l'Enseignement et de la Formation Supérieures

Mechanics 1, standard program for the first semester

Semester: S1

Field: Material Science (SM), **Physics Division**

Specialty: Materials and Physical-Chemical Controls (MCPC)

Level: L1

Professor: Said BENRAMACHE

Lesson No. 1 of 15

2023/2024

Mechanics 1, standard program for the first semester

In mechanics 1 for the first semester, we will studying the laws of classical mechanics, which its four chapters:

- ✓Chapter One: Vector Calculus.
- ✓Chapter Two: Kinetics.
- ✓Chapter Three: Newton’s Laws.
- ✓Chapter Four: Work and energy.

قائمة المراجع التي يمكن الإستعانة بها في مقياس فيزياء 1

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2. د. فيزازي أحمد، ميكانيك النقطة المادية،ملخص للدروس و 100 تمرين محلول، ديوان المطبوعات الجامعية، الجزائر 2008.
3. تيلبي عبد الناصر، ميكانيك النقطة المادية، مطبوعات بيداغوجية، جامعة محمد الصديق بن يحي، جيجل 2014/2015.
4. Alain Gibaud et Michel Henry, « Cours de physique : Mécanique du point » cours et exercices avec solutions , Dunod 1999.
5. Alain Gibaud et Michel Henry, « Cours de physique : Mécanique du point », Dunod 2007.
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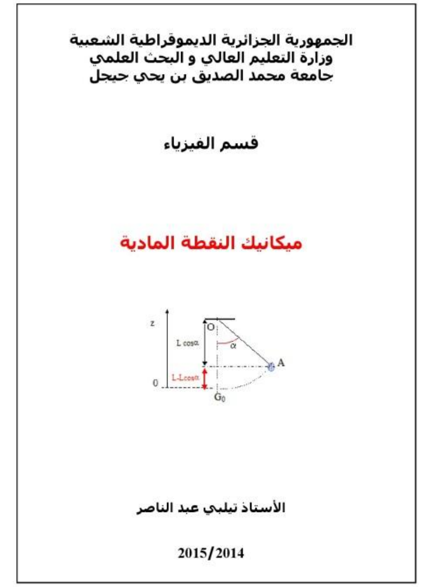
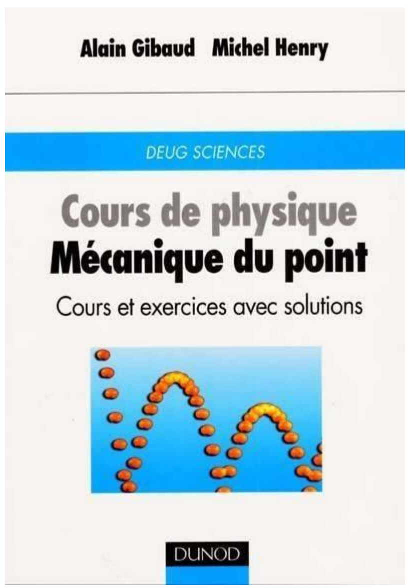

الجمهورية الجزائرية الديموقراطية الشعبية
وزارة التعليم العالي و البحث العلمي
جامعة محمد الصديق بن يحي جيجل

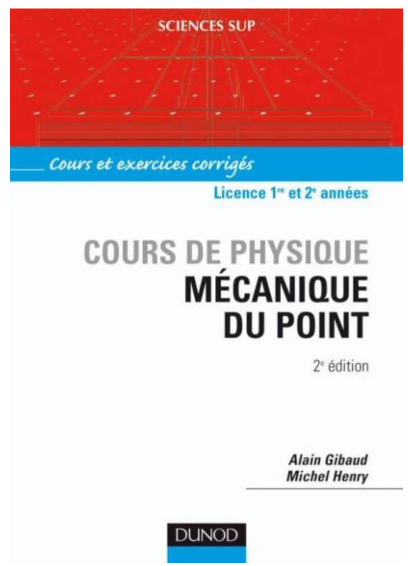
قسم الفيزياء

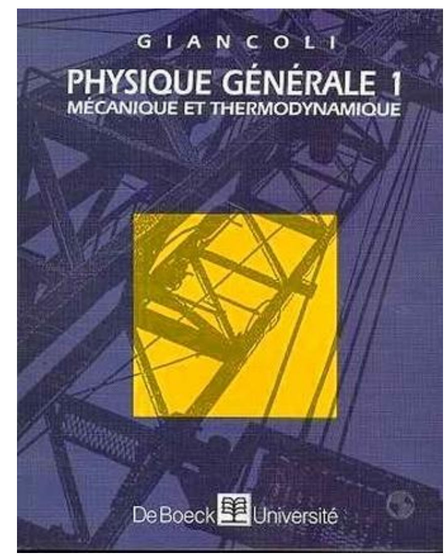
ميكانيك النقطة المادية

الأستاذ تيلبي عبد الناصر

2015/2014







Chapter One: Vector Calculus.

The aim of this chapter is to study the vectors represented for vector calculus, the scalar product, and the radial product, and we show the properties of each them by studying them in two or three dimensions..

1.Introduction

Scalar Quantities are defined as the physical quantities that have magnitude or size only. For example, Mass, Volume ,Temperature, etc

However, vector quantities are those physical quantities that have both magnitude and direction like displacement, velocity, acceleration, force, Electric Charge, etc.

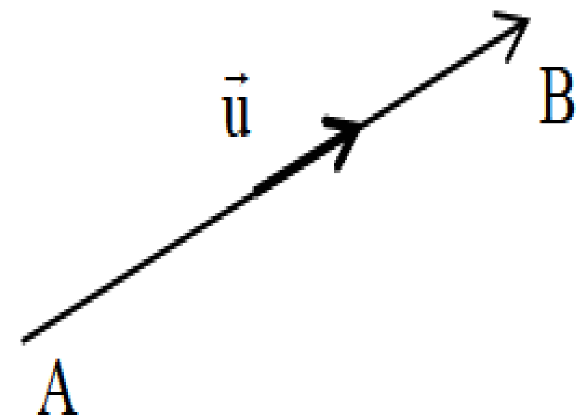
Scale magnitude and scale magnitude are distinguished by a number ("density") and a unit.

Example 1: Mass is expressed in kilograms (kg): M=60kg

Example 1: The vector quantities is determined by a unit vector and its magnitude

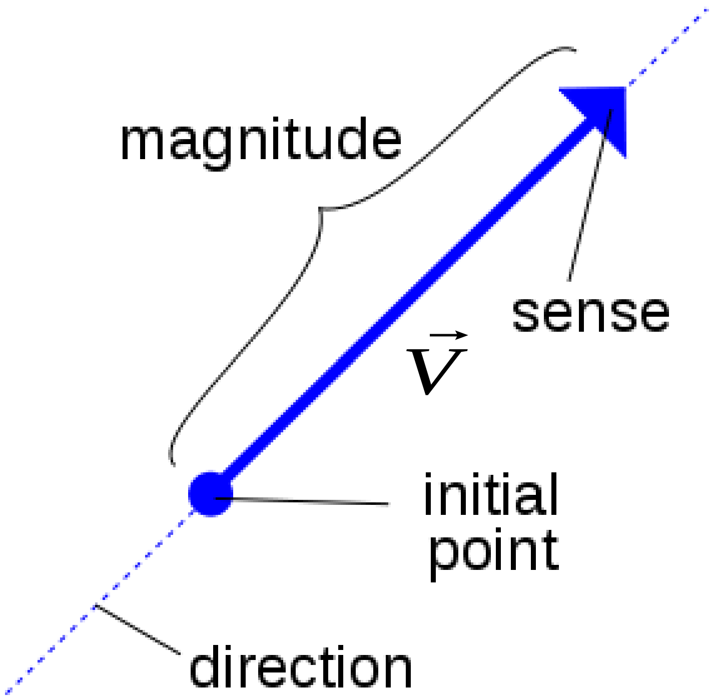
$$\overrightarrow{AB} = AB\vec{u}$$

$$\|\vec{u}\| = 1$$



2.Vector definition

Vector is represented by directed line segment where length of line gives the Magnitude of vector.

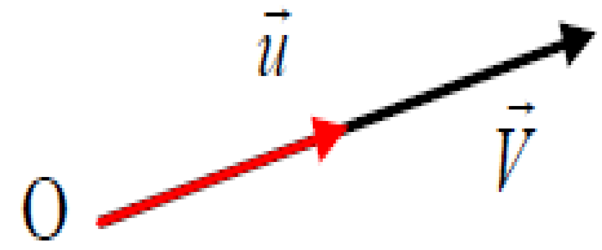


$$\left\|\overrightarrow{MN}\right\| = MN$$

$$\left\|\vec{V}\right\| = V.$$

3. Unit vector

A unit ray is a magnitude ray equal to one (i.s. the number one).



$$\vec{V} = \vec{u}V = V\vec{u}$$

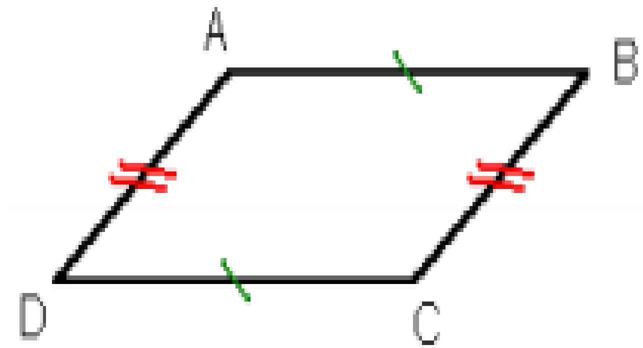
Note:

Two vectors are equal if they have the same direction, the same inclination, and the same magnitude.

Examples

Example 1:

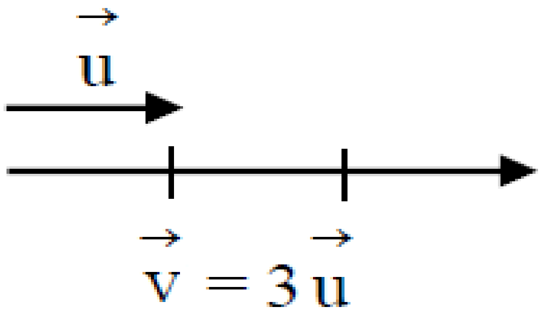
In parallelogram ABCD we get the following:



$$\vec{AB} = \vec{DC} \qquad \vec{AD} = \vec{BC} \qquad \vec{BA} = \vec{CD} \qquad \vec{DA} = \vec{CB}$$

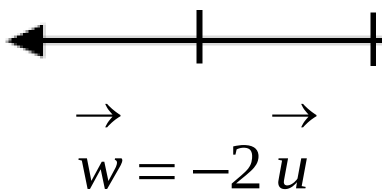
Example 2:

Let $\vec{u}$ be a unit vector and *k* is real number non-zero



If k = 3

$$\vec{v} = k\vec{u} = 3\vec{u}$$



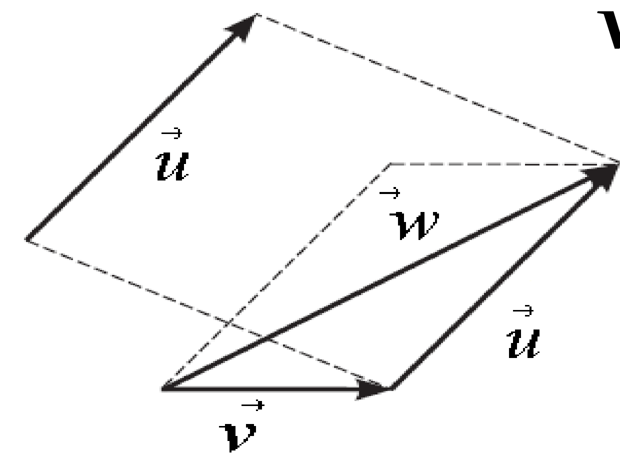
If k = – 2

$$\vec{w} = k\vec{u} = -2\vec{u}$$

4. Vector Operations

$\vec{w}$

4.1. Vector addition



Let $\vec{v}$ and $\vec{u}$ two vectors non zero

$$\vec{w} = \vec{u} + \vec{v}$$

Properties

Commutative

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

$$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$$

Distributive

4.2.The Schall relationship

Three points are given A, B and C

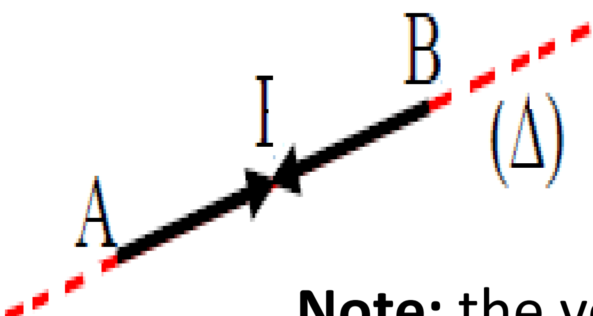
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

Special case:

If the three points are collinear, we get the Schall relation for algebraic measurements:

$$\overline{AC} = \overline{AB} + \overline{BC}$$

Example 1:



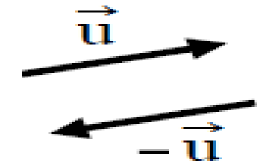
$$\overrightarrow{AB} = \overrightarrow{AI} + \overrightarrow{IB}$$

$$\overline{AB} = \overline{AI} + \overline{IB}$$

$$\overrightarrow{IA} + \overrightarrow{IB} = \vec{0}$$

Note: the vectors $\overrightarrow{IB}$ and $\overrightarrow{IA}$ are the vectors with the same length and opposite in the direction

Example 2:



Let $\vec{u}$ be a vector and *k* is real number non-zero

$$\vec{u} + \left(-\vec{u}\right) = \vec{0}$$

Example 3:

Let $\vec{u}$ and $\vec{v}$ two vectors t vector and *k* and *k' are* real numbers non-zeros

1

$$k\left(\vec{u} + \vec{v}\right) = k\vec{u} + k\vec{v}$$

$$-3\left(\vec{u} + \vec{v}\right) = -3\vec{u} - 3\vec{v}$$ أو $$2\left(\vec{u} + \vec{v}\right) = 2\vec{u} + 2\vec{v}$$

2

$$\left(k + k'\right)\vec{u} = k\vec{u} + k'\vec{u}$$

$$\left(2 + 3\right)\vec{u} = 2\vec{u} + 3\vec{u} = 5\vec{u}$$

3 $(k \times k')\vec{u} = k(k'\vec{u})$

$\vec{v}$ و $\vec{u}$

$(2 \times 3)\vec{u} = 2(3\vec{u}) = 6\vec{u}$

4 **if** $k\vec{u} = \vec{0}$ **either** $k = 0$ **or** $\vec{u} = \vec{0}$

Note:

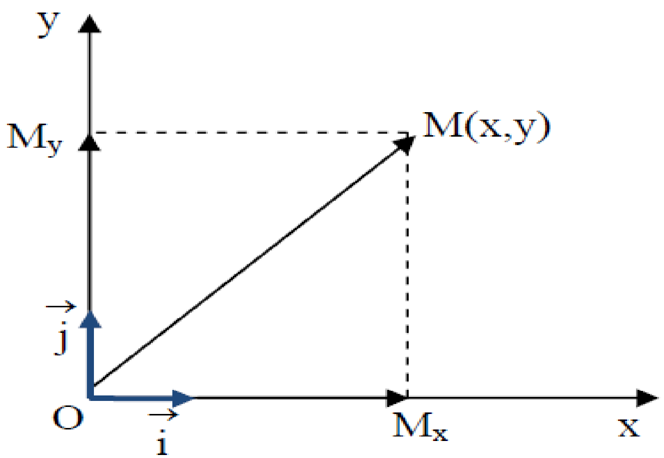
If the vectors $\vec{u}$ and $\vec{v}$ are parallel so $\vec{v} = k\vec{u}$

Example $\vec{v} = -3\vec{u}$ **or** $\vec{v} = 2\vec{u}$

5. Ve ct or s in R 2 a n d R 3:

5.1. Two-dimensional Cartesian orthogonal system

This system is used to locate a point in a plane. It consists of two axes perpendicular to the plane, Ox and Oy, equipped with two unit vectors and positively oriented
The location of point M on the plane is characterized by the interval Mx and the order My which are the projections of M on the axes Ox and Oy, respectively.
Note



$$\overrightarrow{OM} = \overrightarrow{OM}_x + \overrightarrow{OM}_y$$

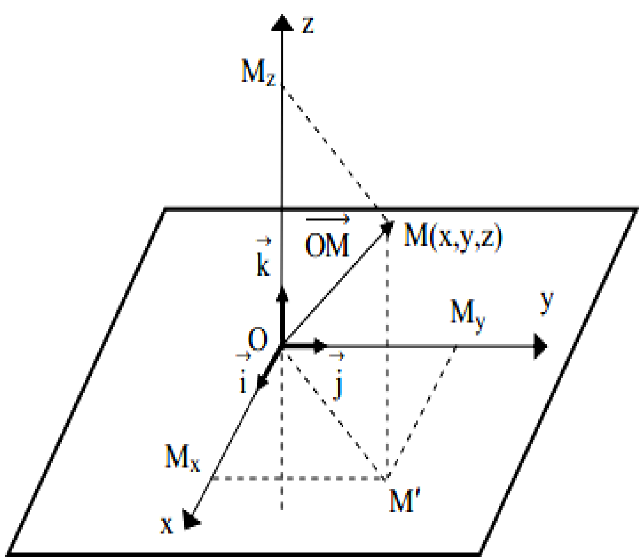
$$\begin{cases} \overrightarrow{OM}_x = x\vec{i} \\ \overrightarrow{OM}_y = y\vec{j} \end{cases}$$

$$\overrightarrow{OM} = x\vec{i} + y\vec{j}$$

The scalar expressions x and y are the Cartesian coordinates of the point M in the system (O,x,y)

5.2. Three-dimensional Cartesian orthogonal system

This system is also used to determine any point M in space (Figure 12. It consists of three axes Ox, Oy and Oz equipped with unit rays and positively oriented. The location of the point M in space is characterized. Mx, My and Mz are the projections of M on the axes Ox and Oy and Oz, respectively. M ′ is the projection of point M on the plane (O,x,y). Note that from Figure 12:



$$\begin{cases} \overrightarrow{OM_x} = x\vec{i} \\ \overrightarrow{OM_y} = y\vec{j} \\ \overrightarrow{OM_z} = z\vec{k} \end{cases}$$

$$\overrightarrow{OM} = \overrightarrow{OM_z} + \overrightarrow{OM'}$$

Note also that from the figure:

$$\overrightarrow{OM} = x\vec{i} + y\vec{j} + z\vec{k}$$

6. The scalar product

Definition: We call the product of vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ the real number: $\overrightarrow{V_1}.\overrightarrow{V_2}$

$$\overrightarrow{V_1}.\overrightarrow{V_2} = V_1.V_2.\cos\left(\overrightarrow{V_1}\overrightarrow{V_2}\right)$$

Example work of force $\overrightarrow{F}$ that causes transmission $\vec{l}$
With the following relationship:

$$W = \overrightarrow{F}.\vec{l} = F.l\cos\alpha$$

where α = $\overrightarrow{F}$ $\vec{l}$ We say that the work is a number real

6.1. Analytical relationship

the two vectors from the (3D) $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ space whose homogeneous orthogonal coordinates $O, \vec{i}, \vec{j}, \vec{k}$ are given by:

$$\overrightarrow{V_1}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \textbf{and} \quad \overrightarrow{V_2}\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

The scalar product

$$\overrightarrow{V_1}.\overrightarrow{V_2} = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\vec{V}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Note 1

$$\overrightarrow{V_1}.\overrightarrow{V_2} = \overrightarrow{V_2}.\overrightarrow{V_1}$$

Note 2 the Magnitude of $\vec{V}$ is $\left\|\overrightarrow{V}\right\| = V = \sqrt{x^2 + y^2 + z^2}$

6.2. Special cases

✓**If** $\overrightarrow{V_1} = \vec{0}$ **and** $\overrightarrow{V_2} = \vec{0}$ **either** $\overrightarrow{V_1}.\overrightarrow{V_2} = 0$

✓**If** $\overrightarrow{V_1} \neq \vec{0}$ **and** $\overrightarrow{V_2} \neq \vec{0}$

if $\overrightarrow{V_1} \perp \overrightarrow{V_2} \Rightarrow \left(\overrightarrow{V_1}, \overrightarrow{V_2}\right) = \frac{\pi}{2} \Rightarrow \cos\frac{\pi}{2} = 0 \Rightarrow \overrightarrow{V_1}.\overrightarrow{V_2} = 0$

if $\overrightarrow{V_1} // \overrightarrow{V_2} \Rightarrow \left(\overrightarrow{V_1}, \overrightarrow{V_2}\right) = 0 \Rightarrow \cos 0 = 1 \Rightarrow \overrightarrow{V_1}.\overrightarrow{V_2} = V_1 V_2$

6.3. Properties of scalar product:

The following three properties of the scalar product are given:

1. The scalar product is commutative

$$\vec{V_1}.\vec{V_2} = \vec{V_2}.\vec{V_1}$$

2. The scalar product is non-distributive for multiplication:

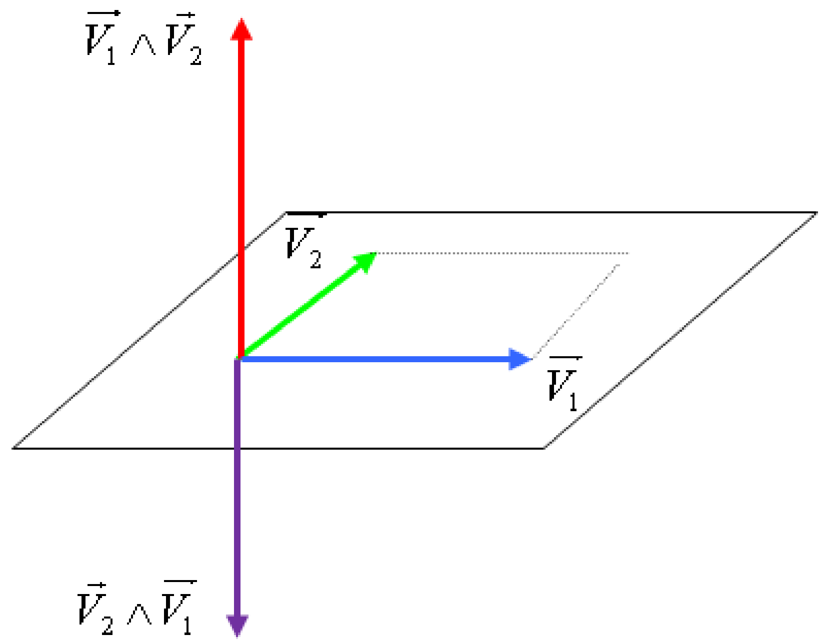
$$\vec{V_1}.(\vec{V_2}.\vec{V_3}) \neq (\vec{V_1}.\vec{V_2}).\vec{V_3}$$

3. The scalar product is distributive for addition:

$$\vec{V_1}.(\vec{V_2} + \vec{V_3}) = \vec{V_1}.\vec{V_2} + \vec{V_1}.\vec{V_3}$$

7. Radial Product

Definition: We know the radial product of the two vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ by the vector $\overrightarrow{V_1} \wedge \overrightarrow{V_2}$ and perpendicular to the plane of the two vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$



Characteristics

Magnitude

$$\left|\vec{V_1} \wedge \vec{V_2}\right| = \left|\vec{V_1}\right| \times \left|\vec{V_2}\right| \times \sin(\vec{V_1}, \vec{V_2}) = V_1 V_2 \sin\theta$$

The direction is determined by the right-hand rule

Note: The expression $\left|\vec{V}_1 \wedge \vec{V}_2\right| = \left|\vec{V}_1\right| \times \left|\vec{V}_2\right| \times \sin(\vec{V}_1, \vec{V}_2) = V_1 V_2 \sin\theta$ represents the area of the parallelogram formed by $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$

Analytical expression of the radial product

Let us have two vectors $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ $\quad \overrightarrow{V_1}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \overrightarrow{V_2}\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

$$\vec{V}_1 \wedge \vec{V}_2 = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \vec{i}\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - \vec{j}\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + \vec{k}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \begin{cases} (y_1 z_2 - z_1 y_2)\vec{i} \\ (z_1 x_2 - x_1 z_2)\vec{j} \\ (x_1 y_2 - y_1 x_2)\vec{k} \end{cases}$$

Magnitude

$$\left|\vec{V}_1 \wedge \vec{V}_2\right| = \sqrt{(y_1 z_2 - z_1 y_2)^2 + (z_1 x_2 - x_1 z_2)^2 + (x_1 y_2 - y_1 x_2)^2} = V_1 V_2 \sin\theta$$

Properties of radial product:

1. The radial product is Anti-commutative

$$\overrightarrow{V_1} \wedge \overrightarrow{V_2} = -\overrightarrow{V_2} \wedge \overrightarrow{V_1}$$

2. The radial product is Non-collective :

$$\overrightarrow{V_1} \wedge (\overrightarrow{V_2} \wedge \overrightarrow{V_3}) \neq (\overrightarrow{V_1} \wedge \overrightarrow{V_2}) \wedge \overrightarrow{V_3}$$

3. The radial product is distributive for addition:

$$\overrightarrow{V_1} \wedge (\overrightarrow{V_2} + \overrightarrow{V_3}) = (\overrightarrow{V_1} \wedge \overrightarrow{V_2}) + (\overrightarrow{V_1} \wedge \overrightarrow{V_3})$$

a result If $\overrightarrow{V_1} \wedge \vec{V_2} = \vec{0}$ **either** $\theta = 0° \Rightarrow \sin\theta = 0$

One of the two vectors are equal to vector zero or the two vectors are parallel

8. Mixed product

The mixed product of three vectors $\overrightarrow{V_1}, \overrightarrow{V_2}$ and $\overrightarrow{V_3}$ it is a real number

$$\vec{V_1}.(\vec{V_2} \wedge \vec{V_3}) = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$= x_1(y_2 z_3 - z_2 y_3) - y_1(z_3 x_2 - x_3 z_2) + z_1(x_2 y_3 - y_2 x_3)$$

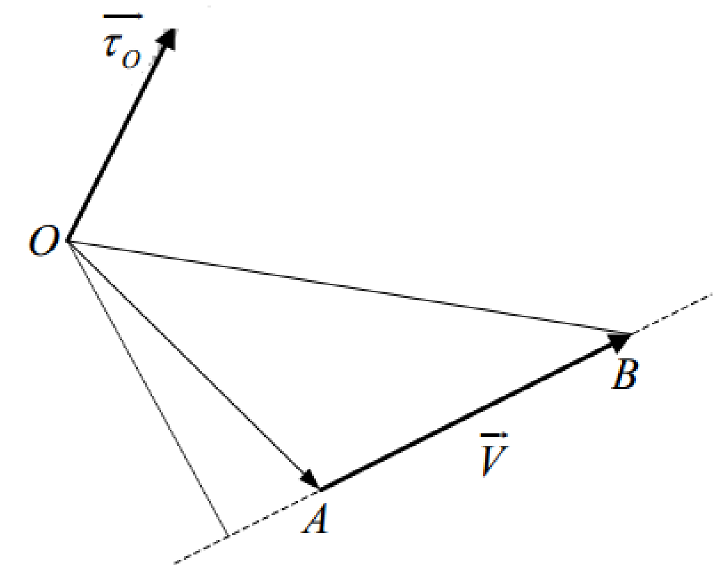
9. The Moment

- **The (Vector torque) moment with respect to a point**

Is defined by

$$\overrightarrow{\tau_0} = \overrightarrow{OA} \wedge \vec{V} = \overrightarrow{OA} \wedge \overrightarrow{AB}$$

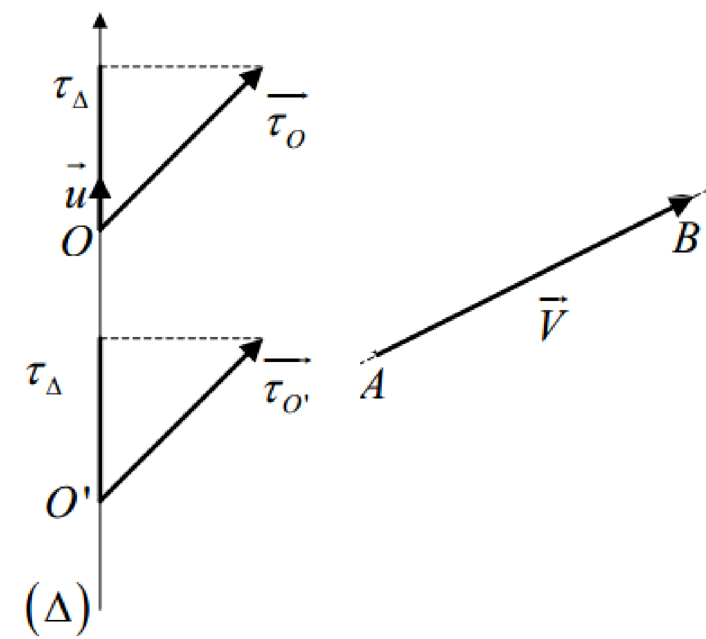
$\left\|\overrightarrow{\tau_0}\right\|$ **It is double the area of the triangle AOB**



- **The moment (torque) relative to the axis**

Is defined by

$$\tau_\Delta = \overrightarrow{\tau_\Delta}.\vec{u} = (\overrightarrow{OA} \wedge \vec{V}).\vec{u}$$



9. Gradation, divergence and rotation

We have the following two functions $f(x, y, z)$ and $\vec{V}(x, y, z)$

$f(x, y, z)$ is a scalar function $\vec{V}(x, y, z)$ is a vector function

We know $\vec{\nabla}$ is the differential radial operator (nabla):

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

where $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$ Are the partial derivatives

- **Gradation**

If it was $f(x,y,z)$ is a scalar function, Its gradient is radial

$$\overrightarrow{grad}\,f = \vec{\nabla}(f) = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

- **Divergence :**

If it was $\vec{V}(V_x, V_y, V_z)$ is a vector function, Its divergence

$$div\vec{V} = \vec{\nabla}.\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

➢ Rotation

If it was $\vec{V}(V_x, V_y, V_z)$ is a vector function, it retion

$$\overrightarrow{zot(\vec{V})} = \vec{\nabla} \wedge \vec{V} = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \left(\frac{\partial V_z}{\partial y} + \frac{\partial V_y}{\partial z} \right)\vec{i} - \left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z} \right)\vec{j} + \left(\frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y} \right)\vec{k}$$

Exercise 1.1

$$[v] = [kt] = [v_0]$$

$$v = \frac{\Delta x}{\Delta t} \qquad [v] = \frac{M}{T} = LT^{-1}$$

$$[v_0] = LT^{-1}$$

$$[v] = [kt] \Rightarrow [kt] = [v]$$

$$\Rightarrow [k] = \frac{[v]}{[t]} = \frac{LT^{-1}}{T} = LT^{-2}$$

Exercise 1.5

$$\vec{V}_2 = -2\vec{i} + 3\vec{j} + 2\vec{k} \qquad \vec{V}_1 = 2\vec{i} - 3\vec{j} + 4\vec{k}$$

$$\vec{V}_1 . \vec{V}_2 = (2\vec{i} - 3\vec{j} + 4\vec{k}).(-2\vec{i} + 3\vec{j} + 2\vec{k}) = (2)(-2) + (-3)(3) + (4)(2) = -4 - 9 + 8 = -5$$

$$\cos(\vec{V}_1 . \vec{V}_2) = \frac{\vec{V}_1 . \vec{V}_2}{V_1 V_2} \qquad \vec{V}_1 . \vec{V}_2 = V_1 V_2 \cos(\vec{V}_1 . \vec{V}_2)$$

$$V_2 = \sqrt{(-2)^2 + (+3)^2 + (+2)^2} = \sqrt{17} \qquad V_1 = \sqrt{(+2)^2 + (-3)^2 + (+4)^2} = \sqrt{29}$$

$$\cos(\vec{V}_1 . \vec{V}_2) = \frac{-5}{\sqrt{29}\sqrt{17}} \Rightarrow \theta\ (\vec{V}_1 . \vec{V}_2) = 103^\circ$$

Exercise 1.7

$$\vec{V}_1 = 3\,\vec{i} - 2\,\vec{j} - 4\,\vec{k} \qquad \vec{V}_2 = -2\,\vec{i} + 2\,\vec{j} + \vec{k} \qquad \vec{V}_3 = 2\,\vec{i} + 3\,\vec{j} - 4\,\vec{k}$$

$$\vec{V}_1 \wedge \vec{V}_2$$

$$(\vec{V}_1 \wedge \vec{V}_2).\vec{V}_1$$

$$\vec{V}_1 \wedge \vec{V}_2 = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ 3 & -2 & -4 \\ -2 & +2 & +1 \end{vmatrix} = \begin{vmatrix} -2 & -4 \\ +2 & +1 \end{vmatrix}\vec{i} - \begin{vmatrix} 3 & -4 \\ -2 & +1 \end{vmatrix}\vec{j} - \begin{vmatrix} 3 & -2 \\ -2 & +2 \end{vmatrix}\vec{k} = +6\vec{i} + 5\vec{j} + 2\vec{k}$$

$$(\vec{V}_1 \wedge \vec{V}_2).\vec{V}_1 = (+6\,\vec{i} + 5\,\vec{j} + 2\,\vec{k}).(+2\,\vec{i} + 3\,\vec{j} - 4\,\vec{k}) = (6)(2) + (5)(3) + (2)(4) = 12 + 15 + 8 = 35$$

Exercise 1.8

$$\overrightarrow{grad}\, f = \vec{\nabla}(f) = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

$$= 2y^2 z\,\vec{i} + 4xyz\,\vec{j} + 2xy^2\,\vec{k}$$

$$div\vec{V} = \vec{\nabla}.\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 2y - z + 0 = 2y - z$$

$$\overrightarrow{zot(\vec{V})} = \vec{\nabla} \wedge \vec{V} = \begin{vmatrix} +\vec{i} & -\vec{j} & +\vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -yz & 3xy \end{vmatrix}$$

$$= \left(\frac{\partial(3xy)}{\partial y} + \frac{\partial(-yz)}{\partial z} \right)\vec{i} - \left(\frac{\partial(3xy)}{\partial x} + \frac{\partial(2xy)}{\partial z} \right)\vec{j} + \left(\frac{\partial(-yz)}{\partial x} + \frac{\partial(2xy)}{\partial y} \right)\vec{k}$$

$$= (3x - y)\vec{i} - (3y + 0)\vec{j} + (0 + 2x)\vec{k} \quad = \quad (3x - y)\vec{i} - 3y\vec{j} + 2x\vec{k}$$